

# HPCを用いた進化ゲーム理論研究

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# 内容

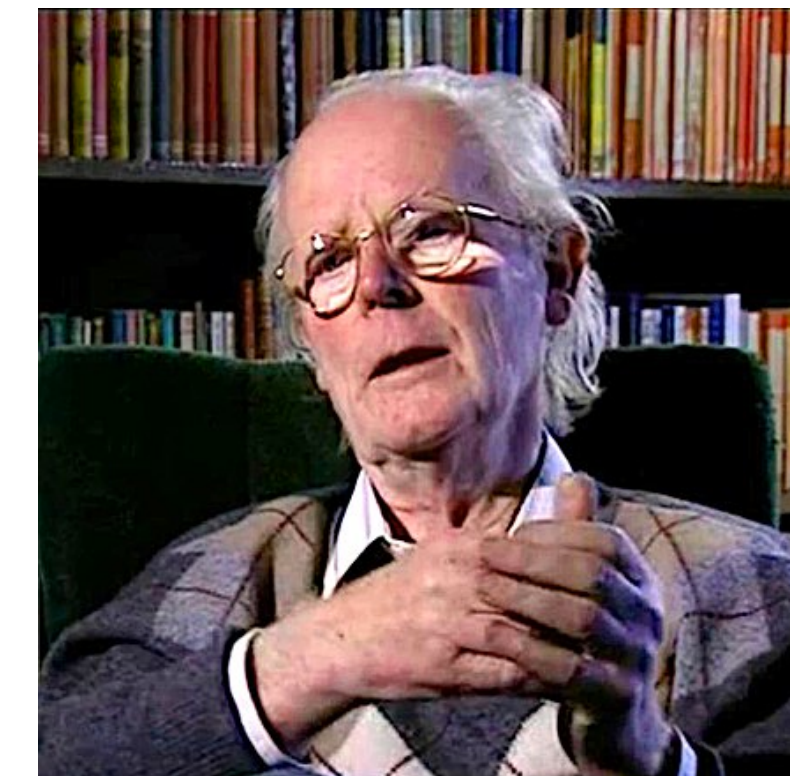
- ゲーム理論、進化ゲーム理論の概要
- 繰り返し囚人のジレンマにおける負けないことが保証された直接互恵戦略
- 今後の展望

# ゲーム理論: 意思決定の科学

- ゲーム理論
  - 複数の意思決定主体の行動を記述する数学的枠組み
  - John von Neumann, Oskar Morgenstern 「ゲームの理論と経済行動」 (1944) が始まり
  - 経済学、社会学、政治学
  - 進化生物学
    - John Maynard Smith 進化ゲーム理論
  - 計算機科学
    - 分散処理、マルチエージェント学習



<http://www.lanl.gov/history/atomicbomb/images/NeumannL.GIF>



CC 表示-継承 3.0

[https://commons.wikimedia.org/wiki/File:John\\_Maynard\\_Smith.jpg](https://commons.wikimedia.org/wiki/File:John_Maynard_Smith.jpg)

# 囚人のジレンマ

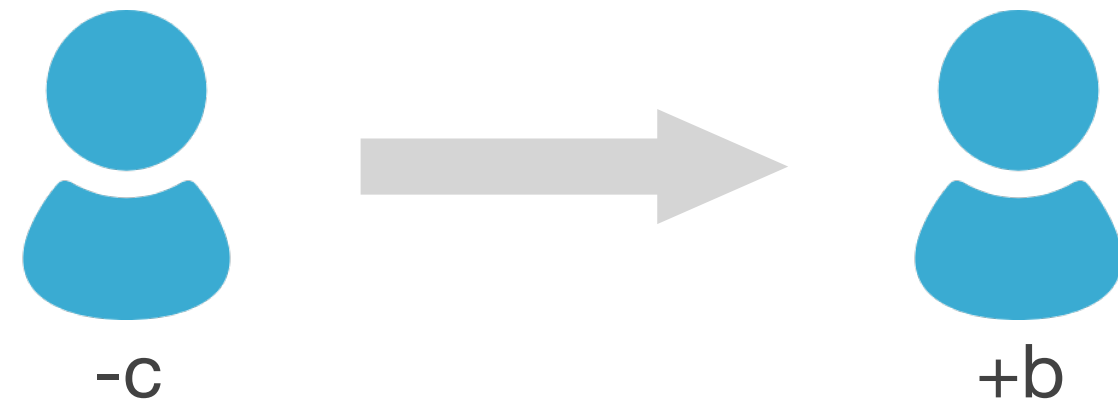
	cooperation	defection
cooperation	(3,3)	(0,5)
defection	(5,0)	(1,1)



- 戦略: 協力、裏切りをプレイヤーが選択する
- 互いに裏切りあう状況が唯一のNash均衡
  - 自分が行動を変えるインセンティブがない安定した状態（物理系でいうと局所安定状態みたいなもの）
- 個人の利得の最適化 ≠ 社会全体の利得の最適化
  - 社会的ジレンマ、コモンズの悲劇、合成の誤謬
  - 地球温暖化、自然破壊、乱獲、過放牧、公共の施設の利用、買い占め、パンデミック下での行動変容

# 協力行動の進化

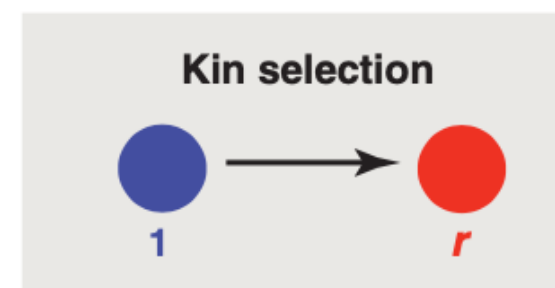
- 協力行動：コストを払って、相手に利益を与える ( $b > c$ )
- なぜ協力行動が進化した？



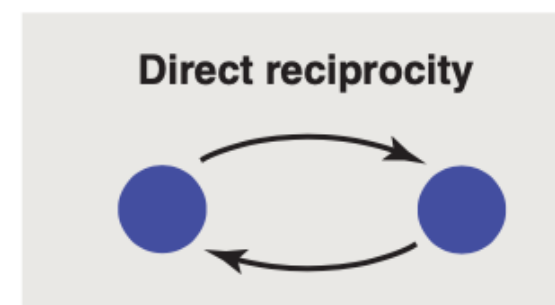
$$\begin{array}{l}
 \text{cooperate(C)} \\
 \text{defect(D)}
 \end{array}
 \begin{pmatrix}
 & C & D \\
 C & b - c & -c \\
 D & b & 0
 \end{pmatrix}$$

## Five Rules for the Evolution of Cooperation

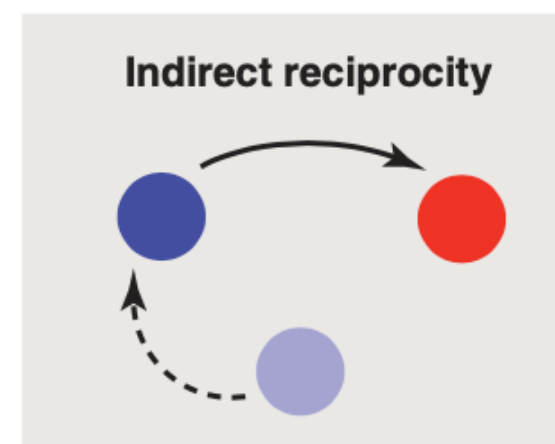
Martin A. Nowak



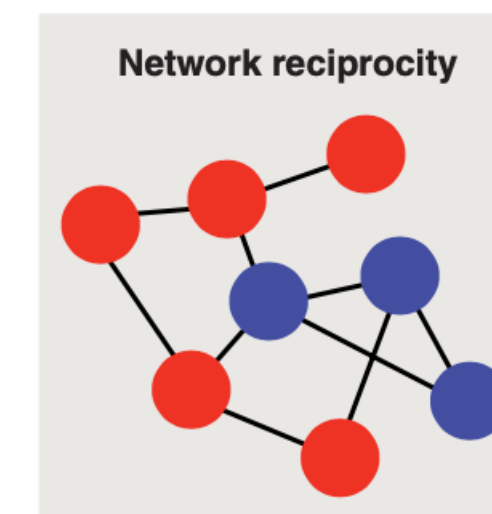
近親選択



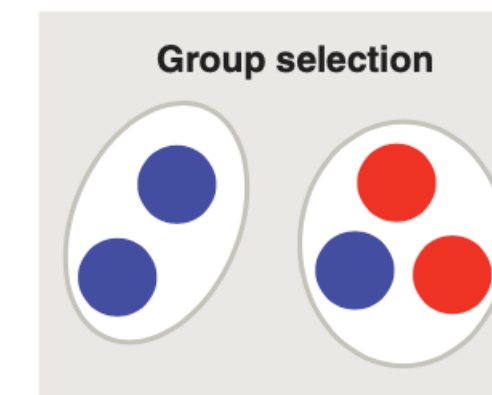
直接互惠



間接互惠



ネットワーク（空間）互惠



グループ選択

● Cooperators ● Defectors

# 内容

- ゲーム理論、進化ゲーム理論の概要
- 繰り返し囚人のジレンマにおける負けないことが保証された直接互恵戦略
- 今後の展望



# iterated Prisoner's Dilemma

Social Dilemma : social optimum  $\neq$  individual benefit

		player B	
		cooperation	defection
player A	cooperation	(3,3)	(0,5)
	defection	(5,0)	(1,1)



repeated the interaction => cooperation may be a rational choice.

long-term payoff in the repeated game

$$f_i \equiv \lim_{\epsilon \rightarrow 0} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} F_i^{(t)}$$

# well known strategies

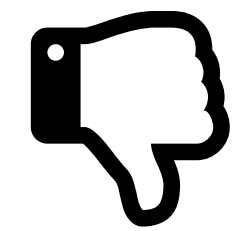
Tit-for-Tat

History	Next move
C,C	C
C,D	D
D,C	C
D,D	D



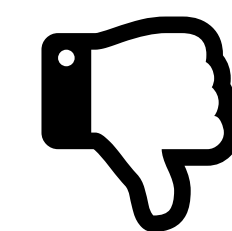
your payoff is no less than the other's

TFT c c c d d c  
c c d d c c



fragile against an error

TFT c c c d c d  
TFT c c d c d c



It cannot exploit naive cooperator.

TFT c c c c c c  
AIC c c c c c c

Win-Stay-Lose-Shift

History	Next move
C.C	C
C.D	D
D.C	D
D.D	C



tolerant against an error

WSLS c c c d c c  
WSLS c c d d c c



It can exploit naive cooperator.

WSLS c c d d d d  
AIC c c c c c c



weak against defectors

WSLS c d c d c d  
AID d d d d d d



# Zero-Determinant Strategy

Press&Dyson PNAS (2012)

## Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent

William H. Press<sup>a,1</sup> and Freeman J. Dyson<sup>b</sup>

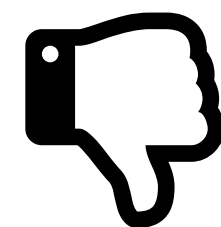
<sup>a</sup>Department of Computer Science and School of Biological Sciences, University of Texas at Austin, Austin, TX 78712; and <sup>b</sup>School of Natural Sciences, Inst for Advanced Study, Princeton, NJ 08540

History	probability of C
C,C	p
C,D	q
D,C	r
D,D	s

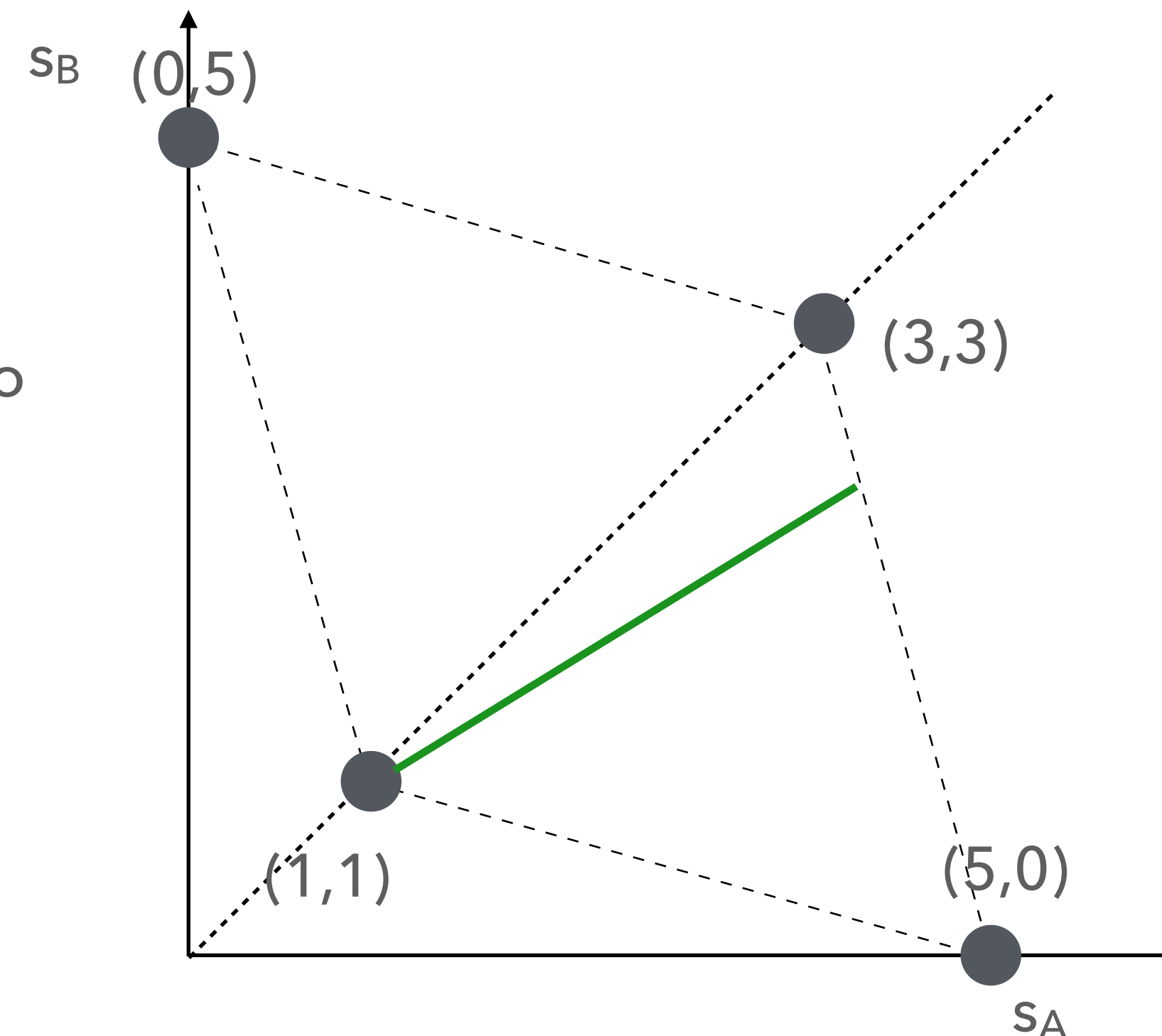
extortionate ZD strategy



It is guaranteed that your payoff is no less than the co-players'.



Cooperation is not maintained.



# longer memory strategies

## Tit-for-2Tat



tolerant against an error

TF2T c c d c c  
 TF2T c c c c c



weak against alternating defector

TF2T c c c c c  
 defector c d c d c

## All-or-Nothing

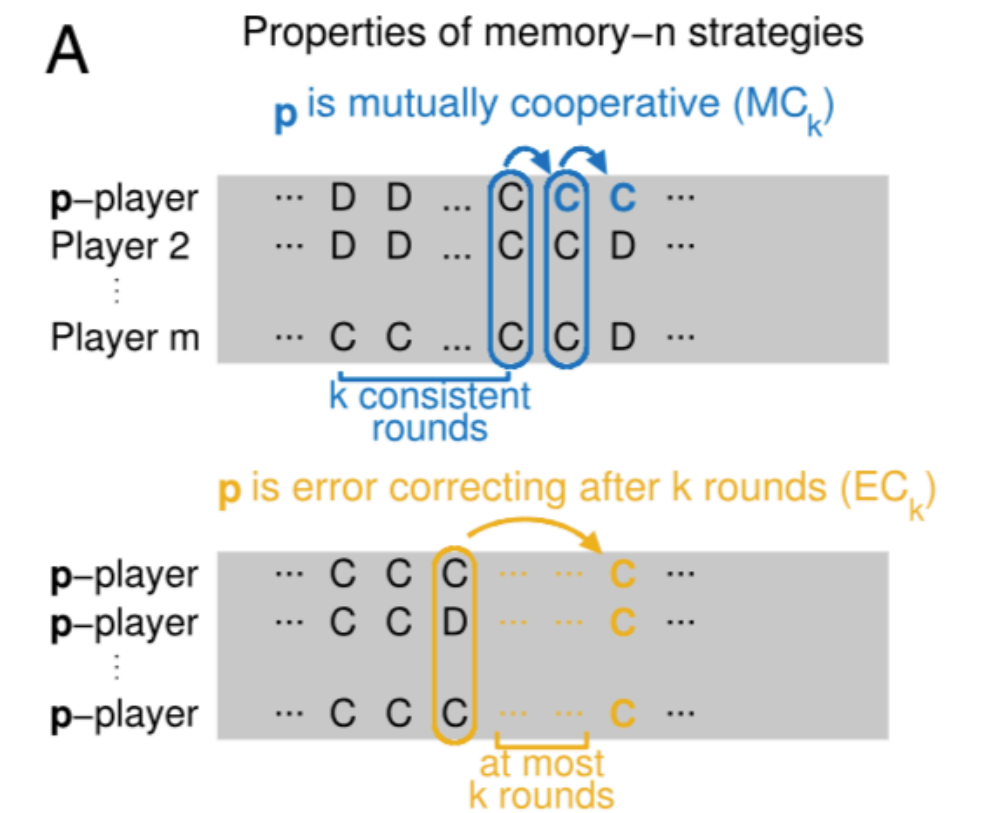
generalization of WSLS



tolerant against an error



Fragile against AllD.



## Memory- $n$ strategies of direct reciprocity

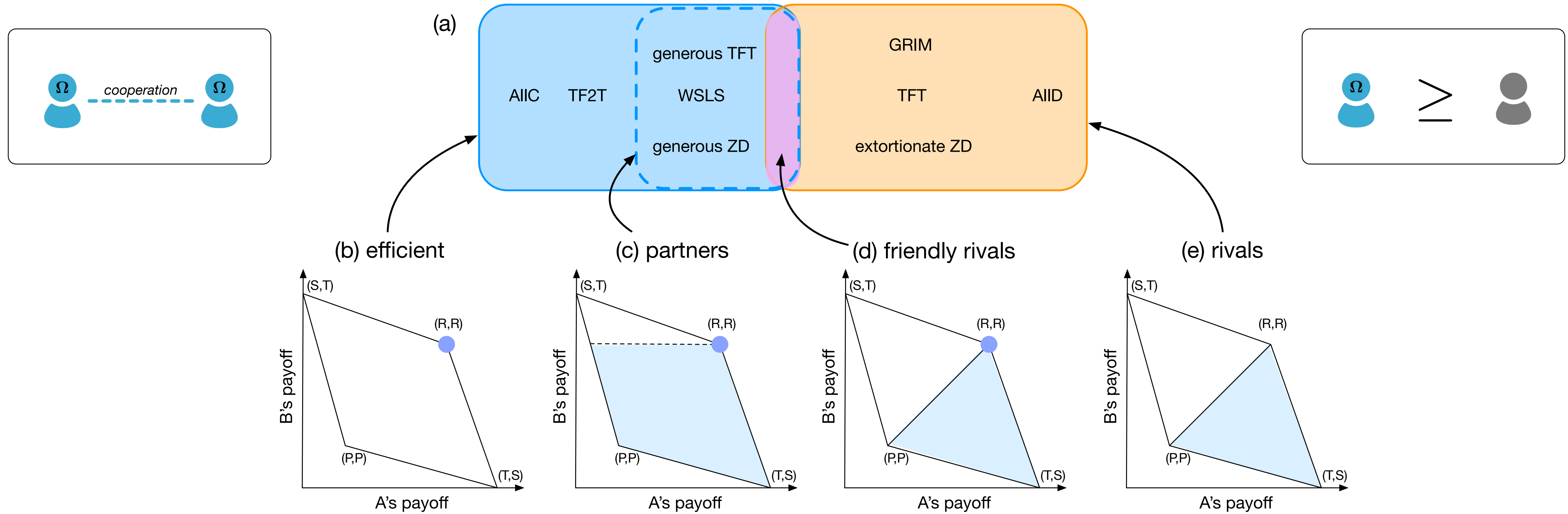
Christian Hilbe<sup>a,1,2</sup>, Luis A. Martinez-Vaquero<sup>b,1</sup>, Krishnendu Chatterjee<sup>a</sup>, and Martin A. Nowak<sup>c,d,e</sup>

Hilbe et al. PNAS(2017)

# partners or rivals

## Partners and rivals in direct reciprocity

Christian Hilbe<sup>1,2\*</sup>, Krishnendu Chatterjee<sup>2</sup> and Martin A. Nowak<sup>1,3</sup>



It would be great if a single strategy works as a **partner as well as a rival.**

## friendly rivals

cooperative Nash equilibrium with a guarantee of never being defeated

# TFT-ATFT: a friendly rival strategy

Yi et al., J. Theor. Biol. (2017)

- # of Memory-1 strategies: 16
  - => no strategy satisfies the criteria
- # of Memory-2 strategies:  $2^{16} = 65536$
- Eight FRs were found.

**Table 1**

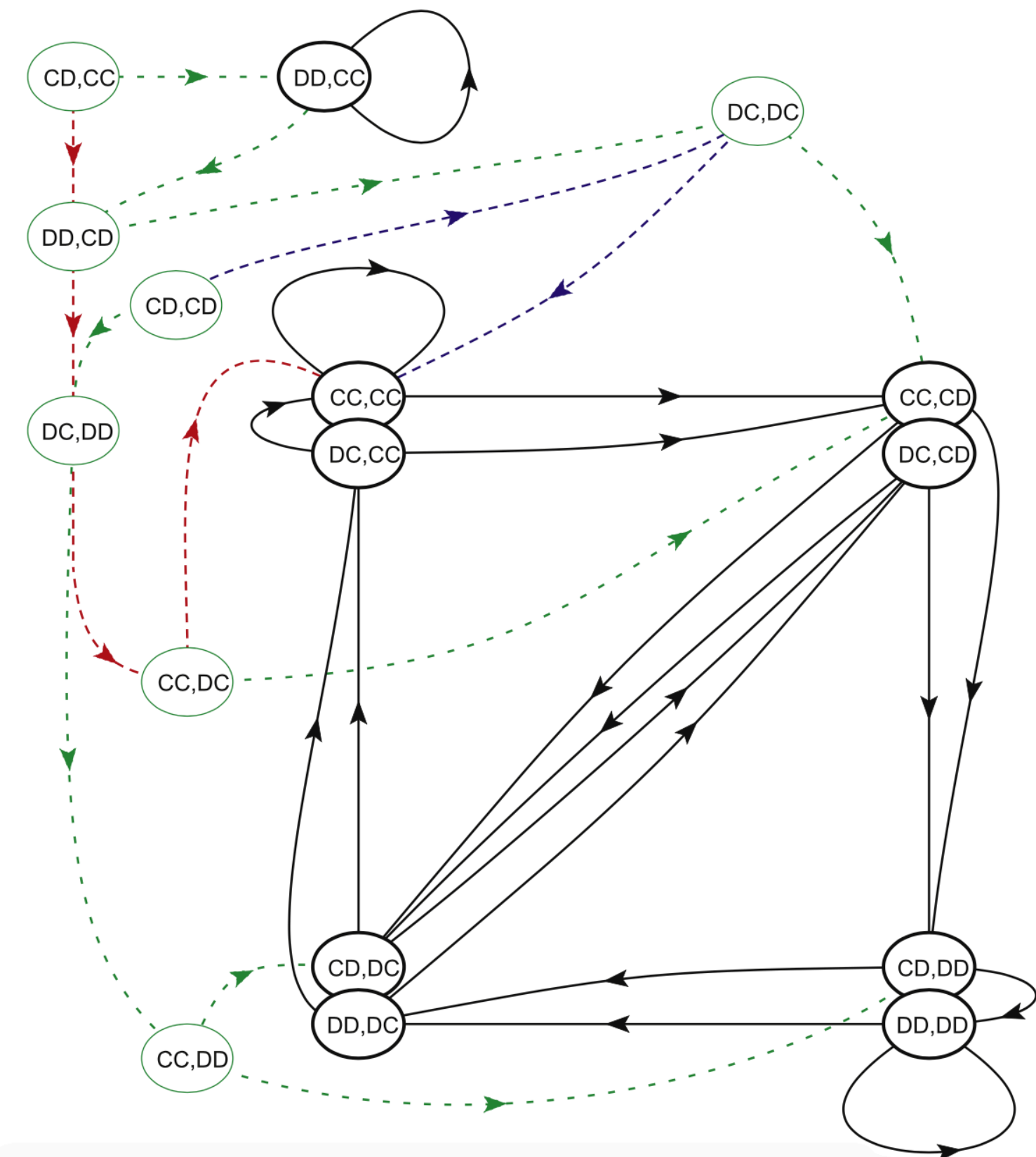
List of moves in TFT-ATFT. This table shows the proposed moves at time  $t$  when the state is given as  $(A_{t-2}A_{t-1}, B_{t-2}B_{t-1})$ , where  $A_{t-2}$  and  $A_{t-1}$  ( $B_{t-2}$  and  $B_{t-1}$ ) are the focal player's (the other player's) moves at the last two steps, respectively. The underlined moves are the same as prescribed by TFT. The states with the dagger symbol are related to the two players' simultaneous mistakes (see text).

State	Move	State	Move
$(CC,CC)$	<u>C</u>	$(DC,CC)$	<u>C</u>
$(CC,CD)$	<u>D</u>	$(DC,CD)$	<u>D</u>
$(CC,DC)$	<u>C</u>	$(DC, DC)^\dagger$	<u>C</u>
$(CC,DD)$	<u>D</u>	$(DC,DD)$	<u>C</u>
$(CD,CC)$	<u>D</u>	$(DD,CC)$	<u>D</u>
$(CD, CD)^\dagger$	<u>C</u>	$(DD,CD)$	<u>C</u>
$(CD,DC)$	<u>C</u>	$(DD,DC)$	<u>C</u>
$(CD,DD)$	<u>D</u>	$(DD,DD)$	<u>D</u>



Combination with anti-tit-for-tat remedies problems of tit-for-tat

Su Do Yi<sup>a</sup>, Seung Ki Baek<sup>b,\*</sup>, Jung-Kyoo Choi<sup>c,\*</sup>





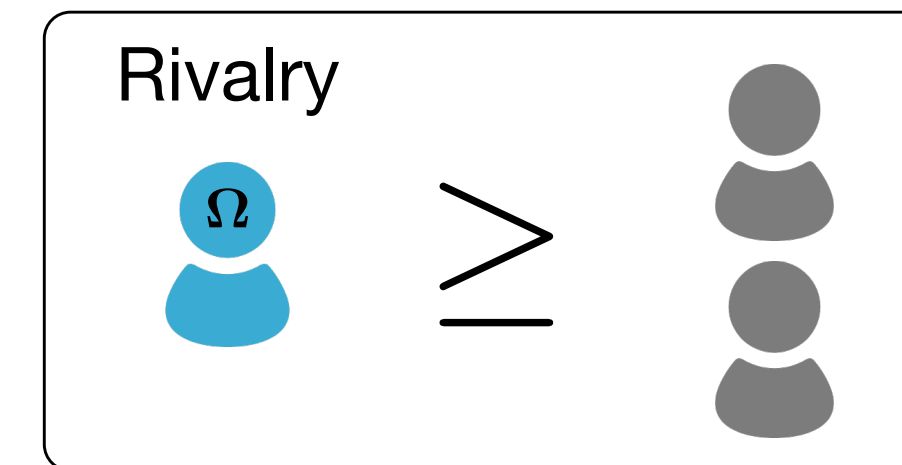
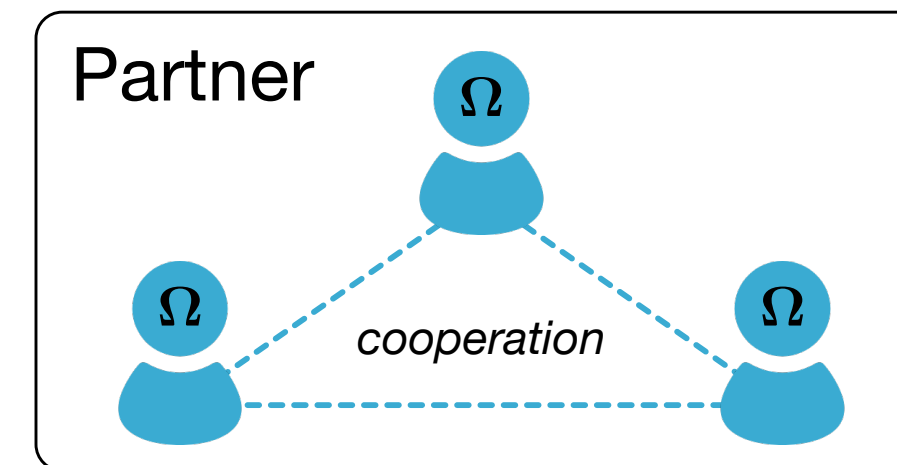
# Games with more than two players?

Y. Murase et al., J. Theor. Biol. (2018)

## 3-person public goods game

payoff matrix

$$M \equiv \left( \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline c & \rho & \frac{2}{3}\rho & \frac{1}{3}\rho \\ d & 1 + \frac{2}{3}\rho & 1 + \frac{1}{3}\rho & 1 \end{array} \right)$$



TFT-ATFT does not work.

$2^{2nm}$  1,099,511,627,776 strategies



# Enumeration of strategies

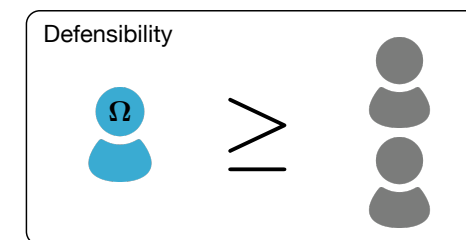
# of m=2 strategies

1,099,511,627,776

Defensibility against AIID

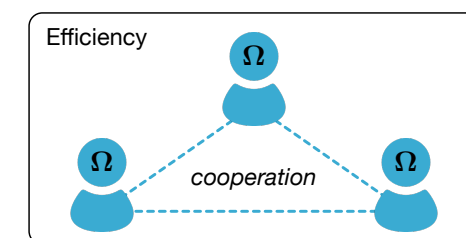
805,306,368

Defensibility



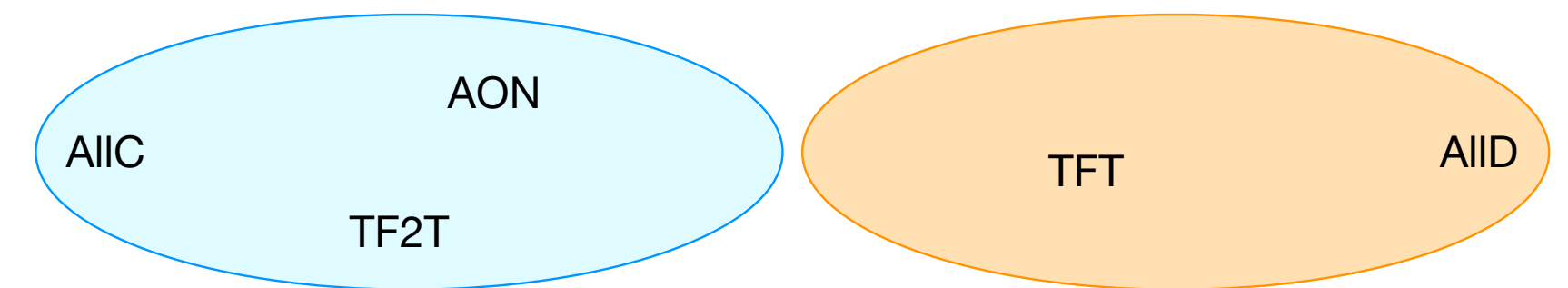
3,483,008

Efficiency



0

**Impossibility : no friendly rival in memory-2 strategies.**



What about memory-**3** strategies?

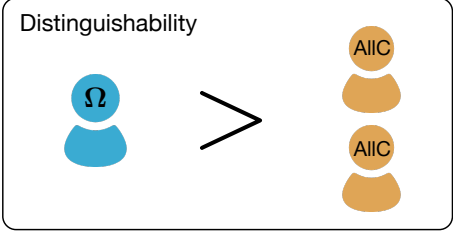
$2^{288}$

497323236409786642155382248146820840100456150797  
347717440463976893159497012533375533056

comparable to the number of protons in the universe

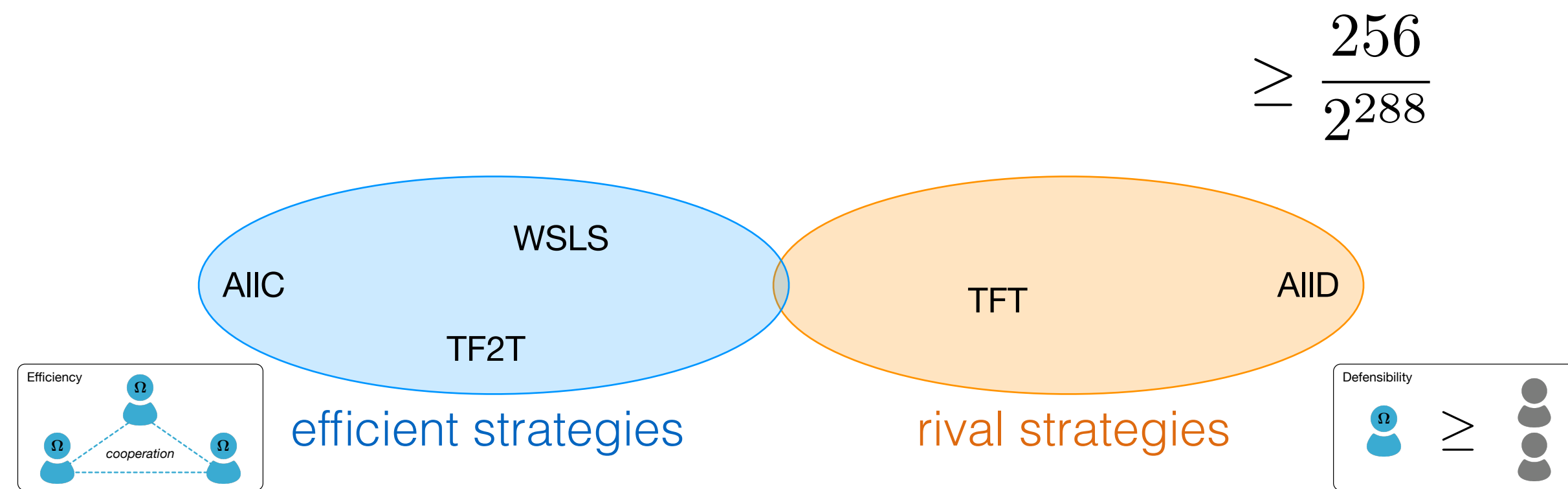


# partially successful strategy

# of m=2 strategies		1,099,511,627,776
Defensibility against AIID		805,306,368
Defensibility		3,483,008
"Partial" Efficiency ( $p_{\text{cooperation}} > 0$ )		544
Distinguishability		<b>256</b>

***"Partially"* friendly rival strategies**





### Friendly rival strategies are found.

Mutual cooperation is reached with probability 1 while keeping the defensibility and the distinguishability.

An example of friendly rival strategies.

**Table 4**

One of successful memory-3 strategies. We have picked up the strategy having the largest number of c. The left column shows the state of Bob and Charlie, whereas Alice's state is shown on the right.

$B_{t-3}B_{t-2}B_{t-1}C_{t-3}C_{t-2}C_{t-1}$	$A_{t-3}A_{t-2}A_{t-1}$							
	ccc	ccd	cdc	cdd	dcc	dcd	ddc	ddd
cccccc	c	c	d	c	c	c	d	c
cccccd/ccdccc	d	c	c	c	d	c	c	c
ccccdc/cdcccc	c	d	c	d	c	d	c	d
ccccdd/cddccc	d	d	d	d	d	d	d	d
cccdcc/dccccc	c	c	d	c	c	c	d	c
cccdcd/dcdccc	d	c	c	c	d	c	c	c
ccddcc/ddcccc	c	d	c	d	c	d	c	d
ccddd/ddddccc	d	d	d	d	d	d	d	d
ccdccc	d	c	c	c	d	c	c	d
cccdc/ccdccc	c	c	c	c	d	c	d	c
cccdcd/cddccc	d	d	d	d	d	d	d	d
ccddcc/dccccc	d	c	c	c	d	c	c	c
ccddcd/dcdccc	d	c	c	d	d	c	c	d
ccdddc/ddcccc	d	c	d	c	d	c	d	c
ccddd/ddddccc	d	d	d	d	d	d	d	d
cdccdc	c	d	c	d	c	c	c	d
cdccdd/cddcdc	d	d	c	d	d	d	c	d
cdccdc/dccccc	c	d	c	c	c	d	c	d
cdcccd/dcdcdc	d	c	d	c	d	c	d	c
cdccdc/ddcccc	c	d	c	d	c	d	c	d
cdccdd/ddddccc	d	d	c	d	d	d	c	d
cddcdd	d	d	c	d	d	d	c	d
cdddcc/dccccc	d	d	d	d	d	d	d	d
cdddcd/dcdccc	d	d	d	d	d	d	d	d
cdddcc/ddcccc	d	d	c	c	d	d	c	d
cdddd/ddddccc	d	d	c	d	d	d	c	d
dccccc	c	c	d	c	c	c	d	c
dcccd/dcdccc	d	c	c	c	d	c	c	c
dccddc/dcdccc	c	d	c	d	c	d	c	d
dccddd/ddddccc	d	d	d	d	d	d	d	d
dcddcd	d	c	c	d	d	c	c	d
dcddd/ddcdcd	d	c	d	c	d	c	d	c
dcddd/ddddccc	d	d	d	d	d	d	d	d
ddccdc	c	d	c	d	c	d	c	d
ddccdd/ddddccc	d	d	c	d	d	d	c	d
dddddd	d	d	c	d	d	d	c	d

# interpretation of m=3 successful strategies

Y. Murase et al., Sci.Rep. (2020)

**Table 4**  
One of successful memory-3 strategies. We have picked up the strategy having the largest number of c. The left column shows the state of Bob and Charlie, whereas Alice's state is shown on the right.

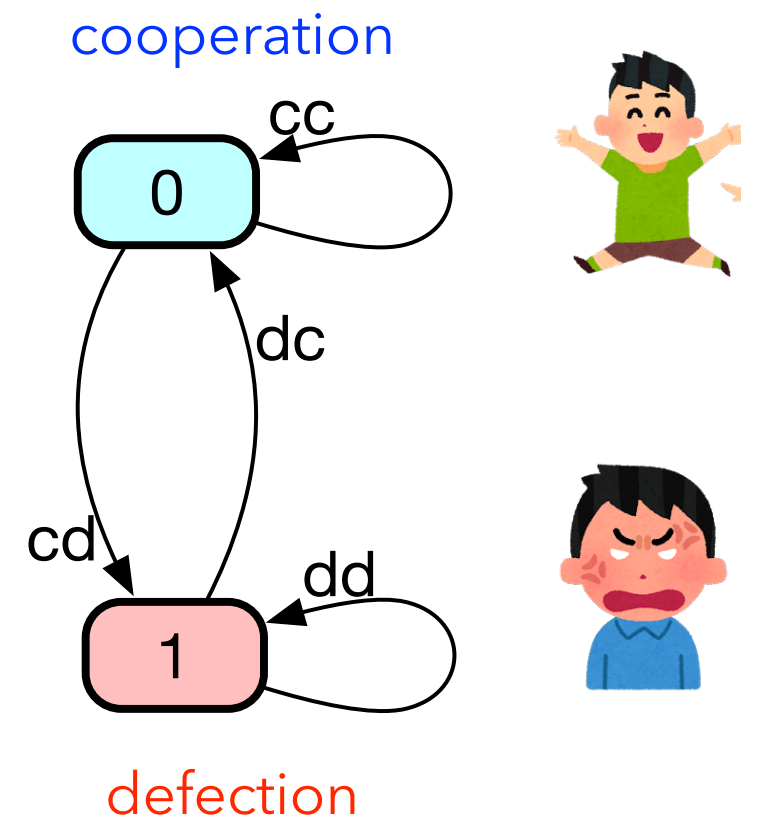
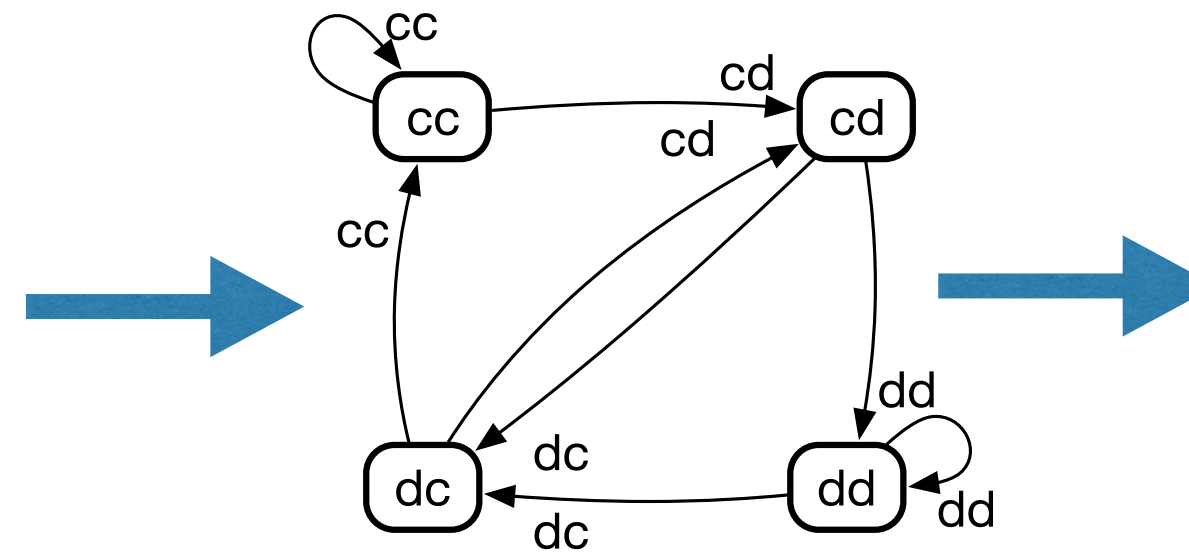
$B_{t-3}B_{t-2}B_{t-1}C_{t-3}C_{t-2}C_{t-1}$	$A_{t-3}A_{t-2}A_{t-1}$							
	ccc	ccd	cdc	cdd	dcc	dcd	ddc	ddd
cccccc	c	c	d	c	c	c	d	c
cccccd/ccdecc	d	c	c	c	d	c	c	c
ccccdc/cdeccc	c	d	c	d	c	d	c	d
ccccdd/cddccc	d	d	d	d	d	d	d	d
cccdec/dccccc	c	c	d	c	c	c	d	c
cccddc/dcdccc	d	c	c	c	d	c	c	c
cccddc/ddcccc	c	d	c	d	c	d	c	d
cccddd/dddccc	d	d	d	d	d	d	d	d
cccdcd	d	c	c	c	d	c	c	d
cccdcc/dcccdd	c	c	c	c	d	c	d	c
cccdcd/cdcdcd	d	d	d	d	d	d	d	d
ccddcc/dccccc	d	d	c	c	d	c	c	c
ccddcd/dcdccc	d	c	c	d	d	c	c	d
ccdddc/ddcccc	d	c	d	c	d	c	d	c
ccdddj/dddccc	d	d	d	d	d	d	d	d
cdccdc	c	d	c	d	c	c	c	d
cdccdd/cddcdc	d	d	c	d	d	d	c	d
cdccdc/dcccdc	c	d	c	c	c	d	c	d
cdccdc/dcdcdc	d	c	d	c	d	c	d	c
cdccdd/dddcdc	d	d	c	d	d	d	c	d
cdccdd	d	d	c	d	d	d	c	d
cdccdc/dcccdd	d	d	d	d	d	d	d	d
cdccdc/dcdcd	d	d	d	d	d	d	d	d
cdccdc/ddccdd	d	d	c	c	d	d	c	d
cdccdd/dddccc	d	d	c	d	d	d	c	d
cdccdd	c	c	d	c	c	c	d	c
dccddc/dcdccc	d	c	c	c	d	c	c	c
dccddc/dddccc	c	d	c	d	c	d	c	d
dccddd/dddccc	d	d	d	d	d	d	d	d
dcdcdc	d	c	c	d	d	c	c	d
dcdcdc/dddccc	d	c	d	c	d	c	d	c
dcdcdc/dddccc	d	d	d	d	d	d	d	d
ddccdc	c	d	c	d	c	d	c	d
ddccdd/dddccc	d	d	c	d	d	d	c	d
ddddd	d	d	c	d	d	d	c	d

an intuitive interpretation is lacking



action table

History	Next move
C,C	C
C,D	D
D,C	C
D,D	D



Core idea: Deterministic Finite Automaton minimization  
 convert a strategy defined by an action table into the equivalent automaton having minimum number of states





# memory length vs the number of players

Y. Murase et al., J. Theor. Biol. (2018)

n=2 : TFT-ATFT (m=2)

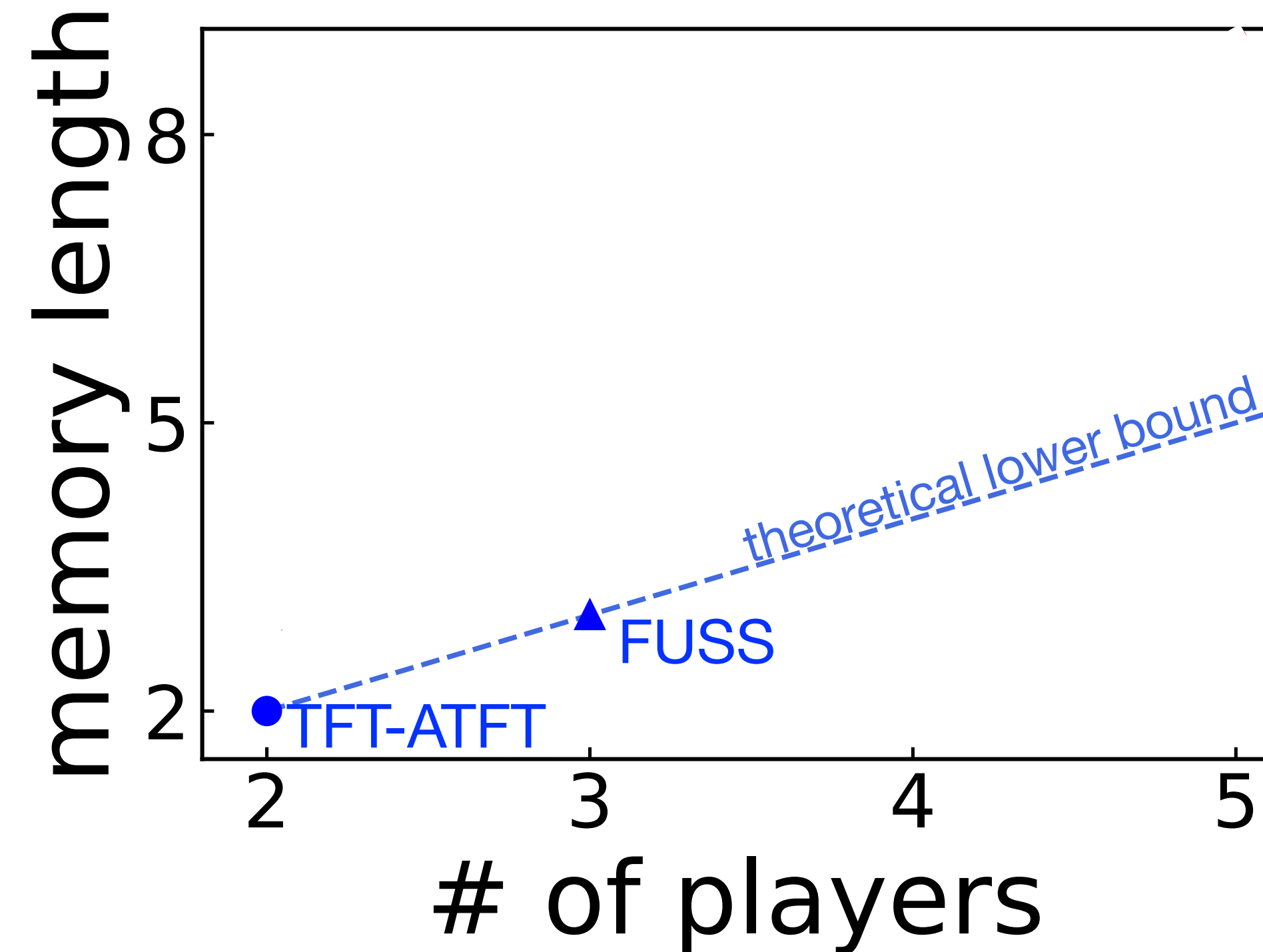
n=3 : memory-3 friendly rival strategies

...

for general n ( $\geq 3$ ), we show

$$m \geq n$$

There is a critical memory length above which friendly rivals may exist.

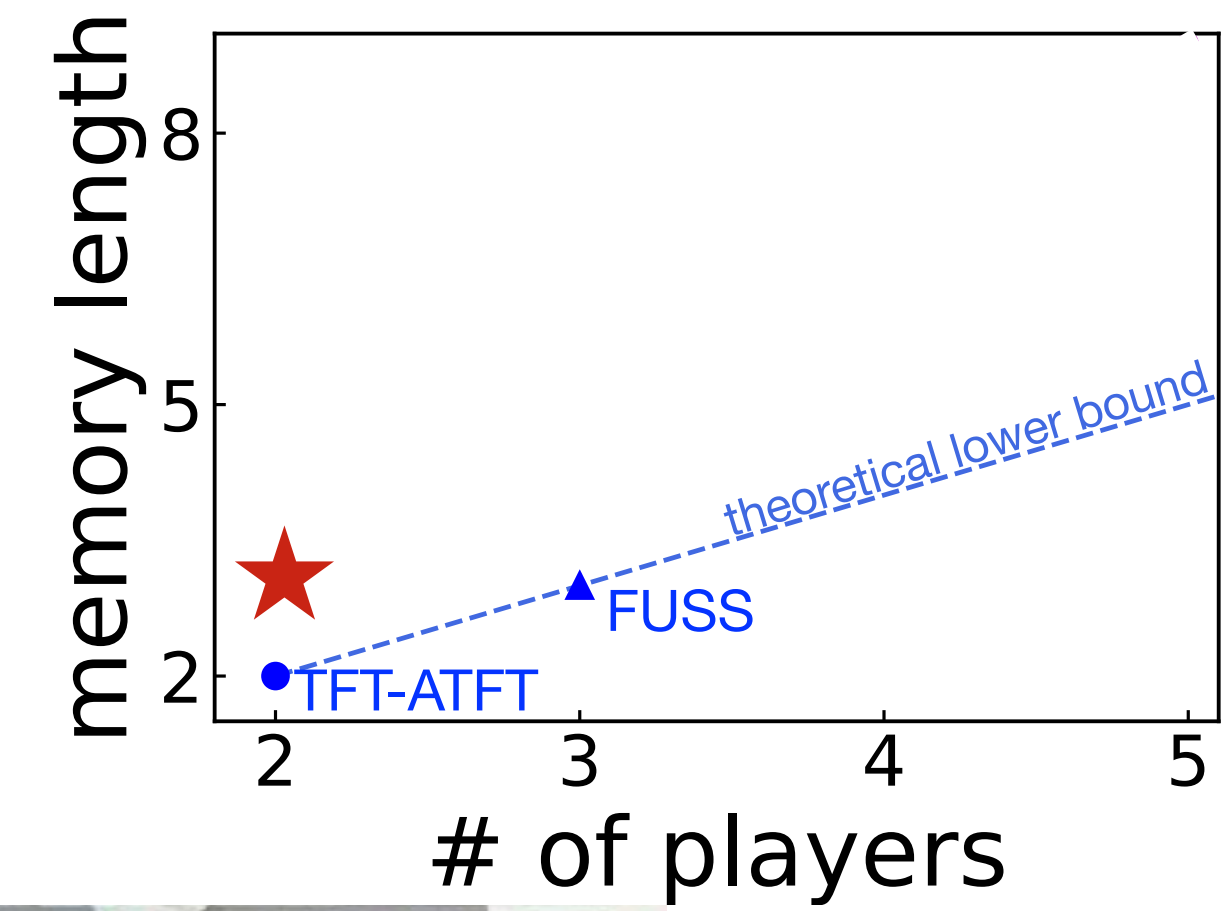




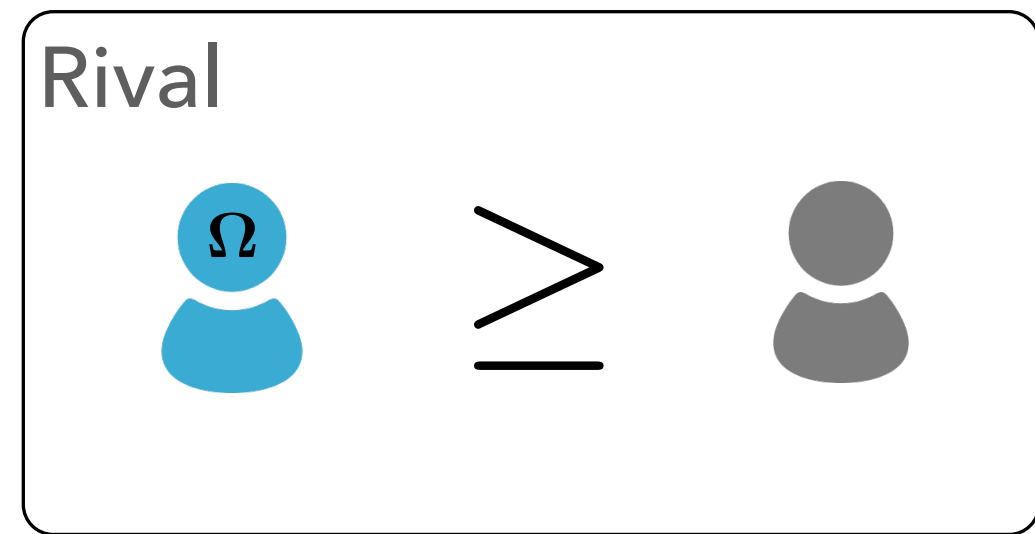
# two-player memory-three strategies

# of memory- $m$  strategies:  $2^{2^{2^m}}$

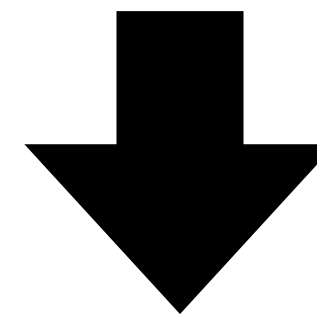
$m=1$	: 0 / 256
$m=2$	: 4 / 65,536
$m=3$	: ? / 18,446,744,073,709,551,616



# checking rivalry

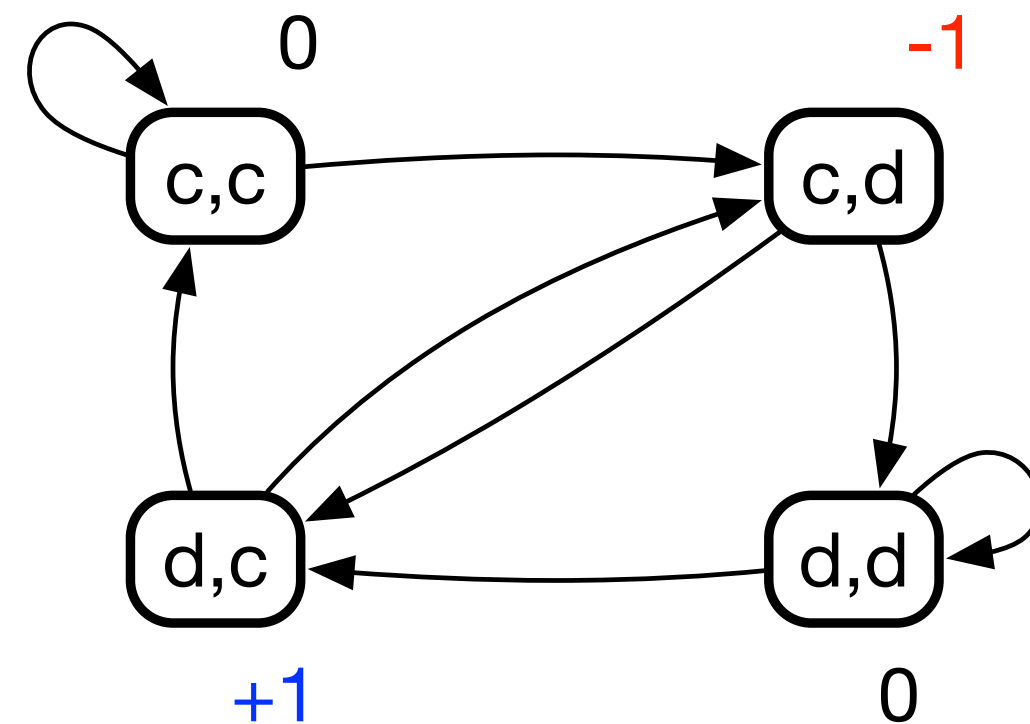


It is guaranteed that the long-term payoff is no less than any of co-players' payoff.

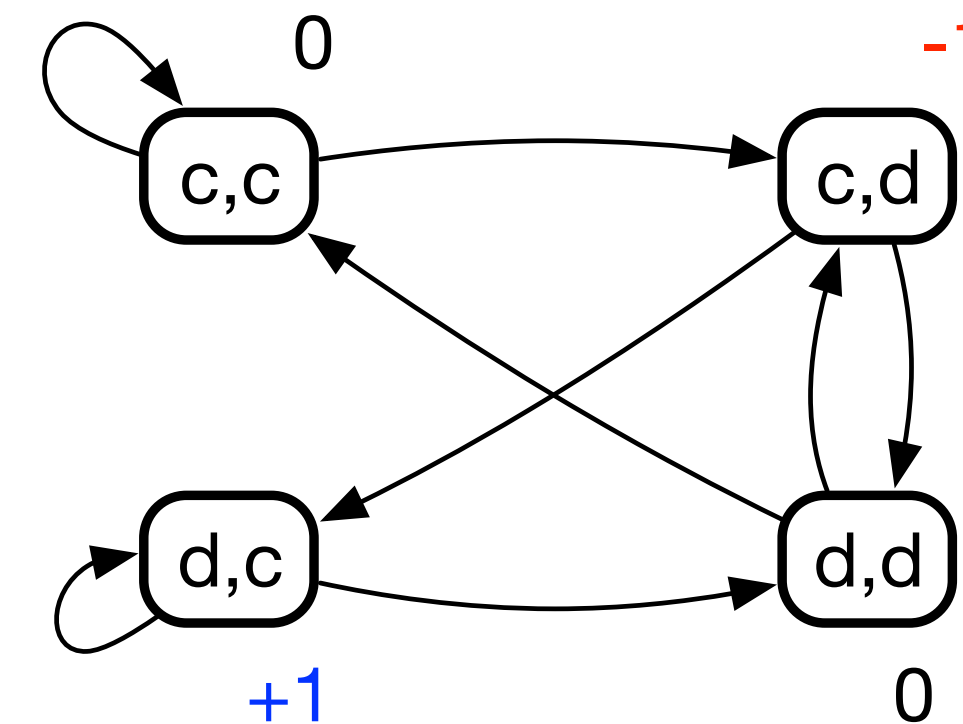


existence of a negative cycle in the state transition graph

e.g. TFT strategy in 2-person game

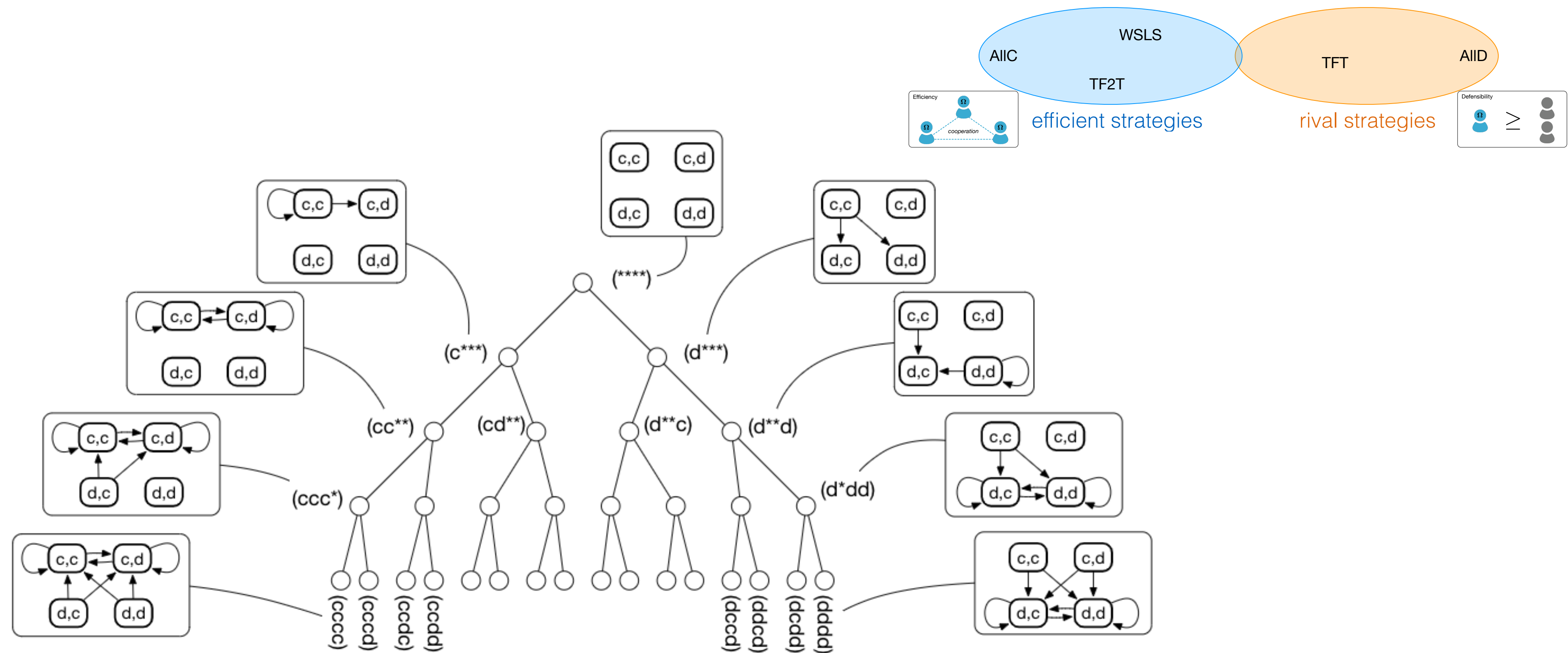


e.g. WSLS strategy in 2-person game



Floyd-Warshall algorithm



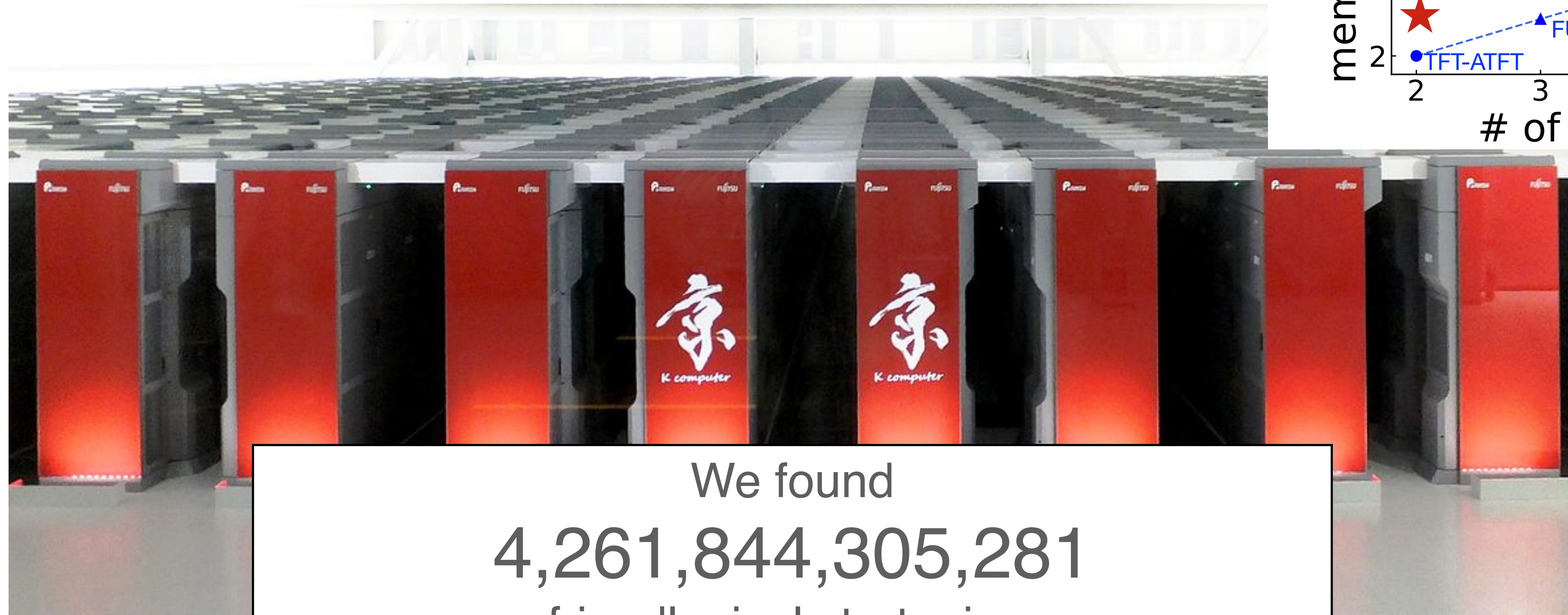
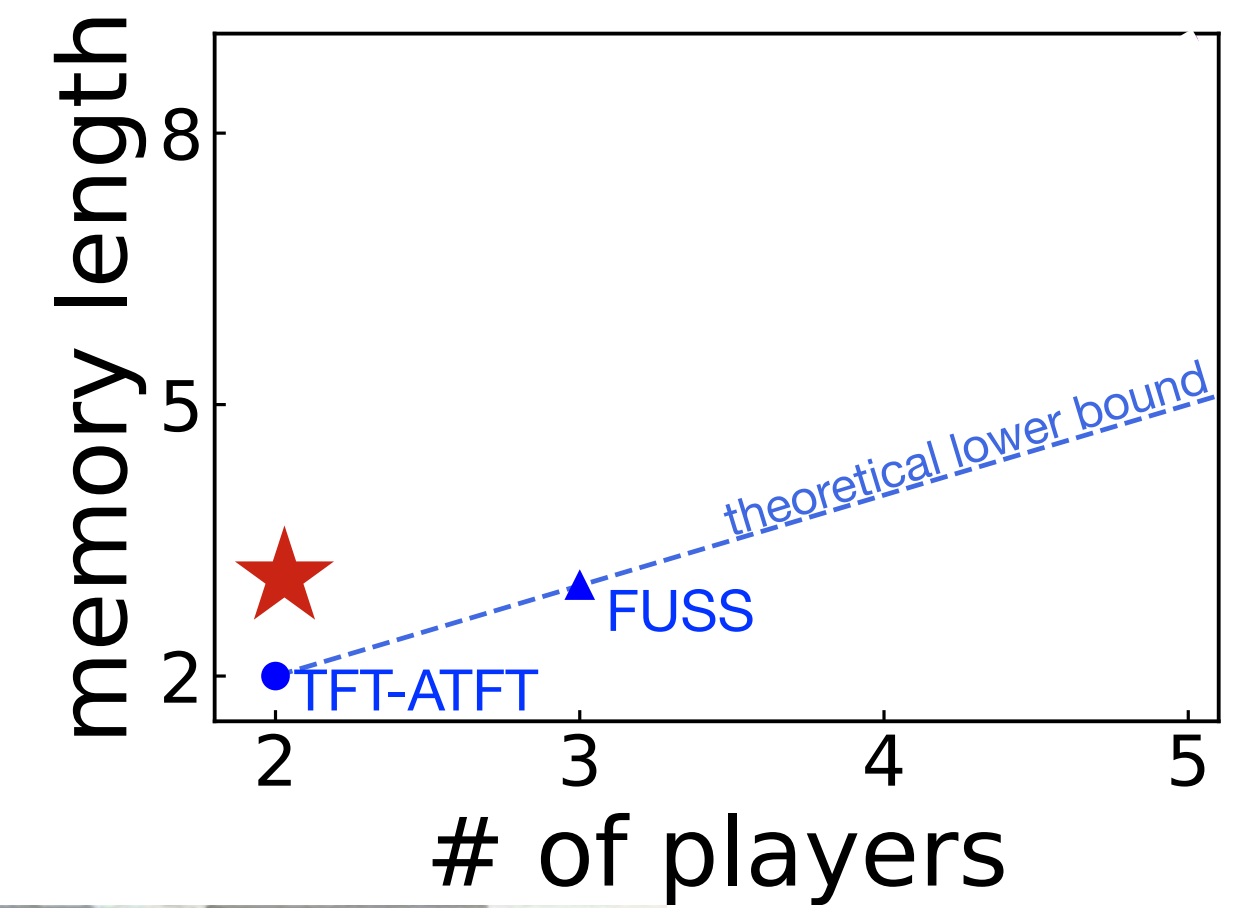


戦略を完全二分木で表現する。葉が戦略。ノードが戦略の共通部分に対応  
 できるだけ早く枝刈りができるように、rivalryとefficiencyを同時にチェックする  
 二分木を作るときの順番（どのビットから固定するか）も重要

# two-player memory-three strategies

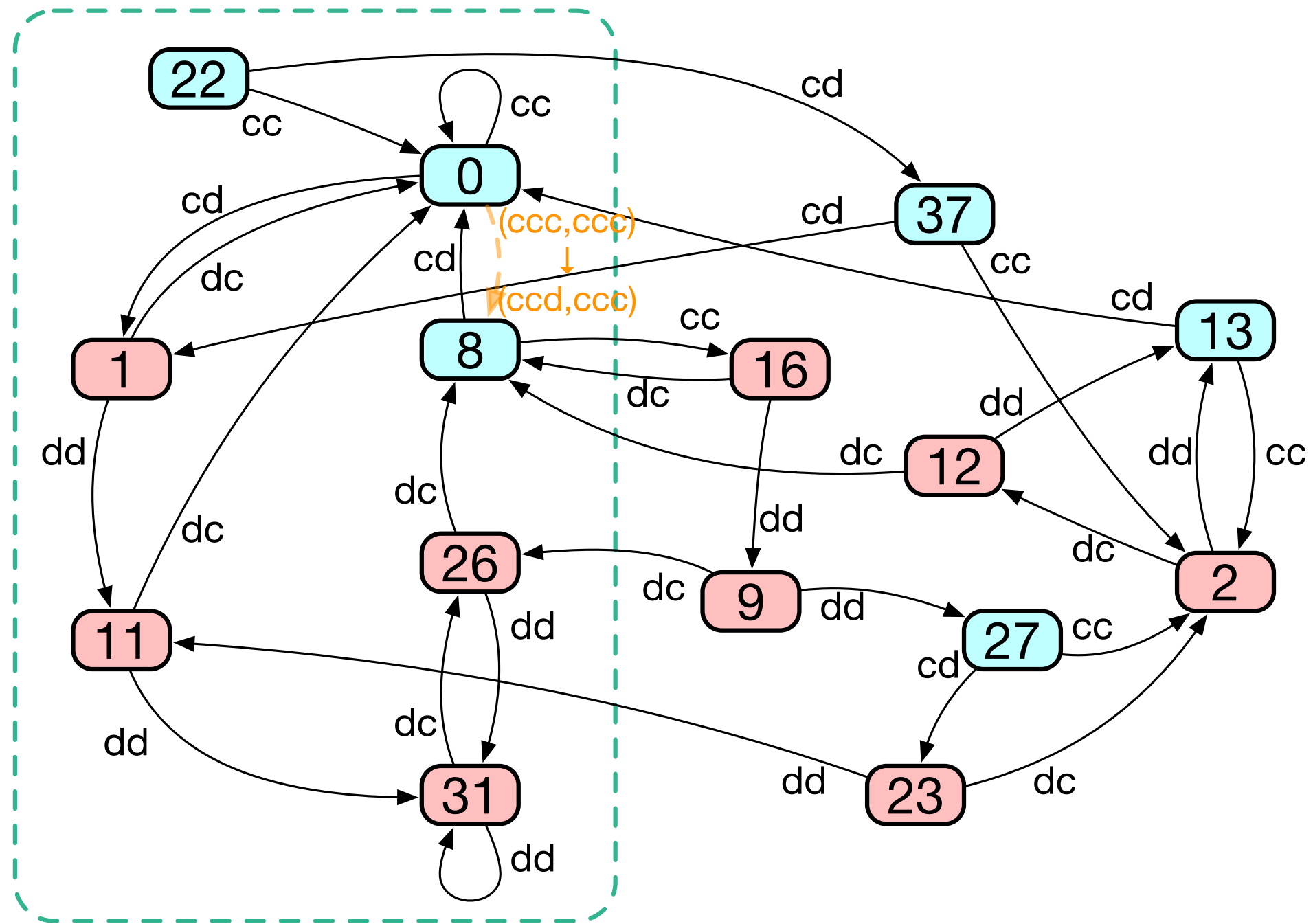
# of memory- $m$  strategies:  $2^{2^{2m}}$

$m=1$	: 0 / 256
$m=2$	: 4 / 65,536
$m=3$	: ? / 18,446,744,073,709,551,616

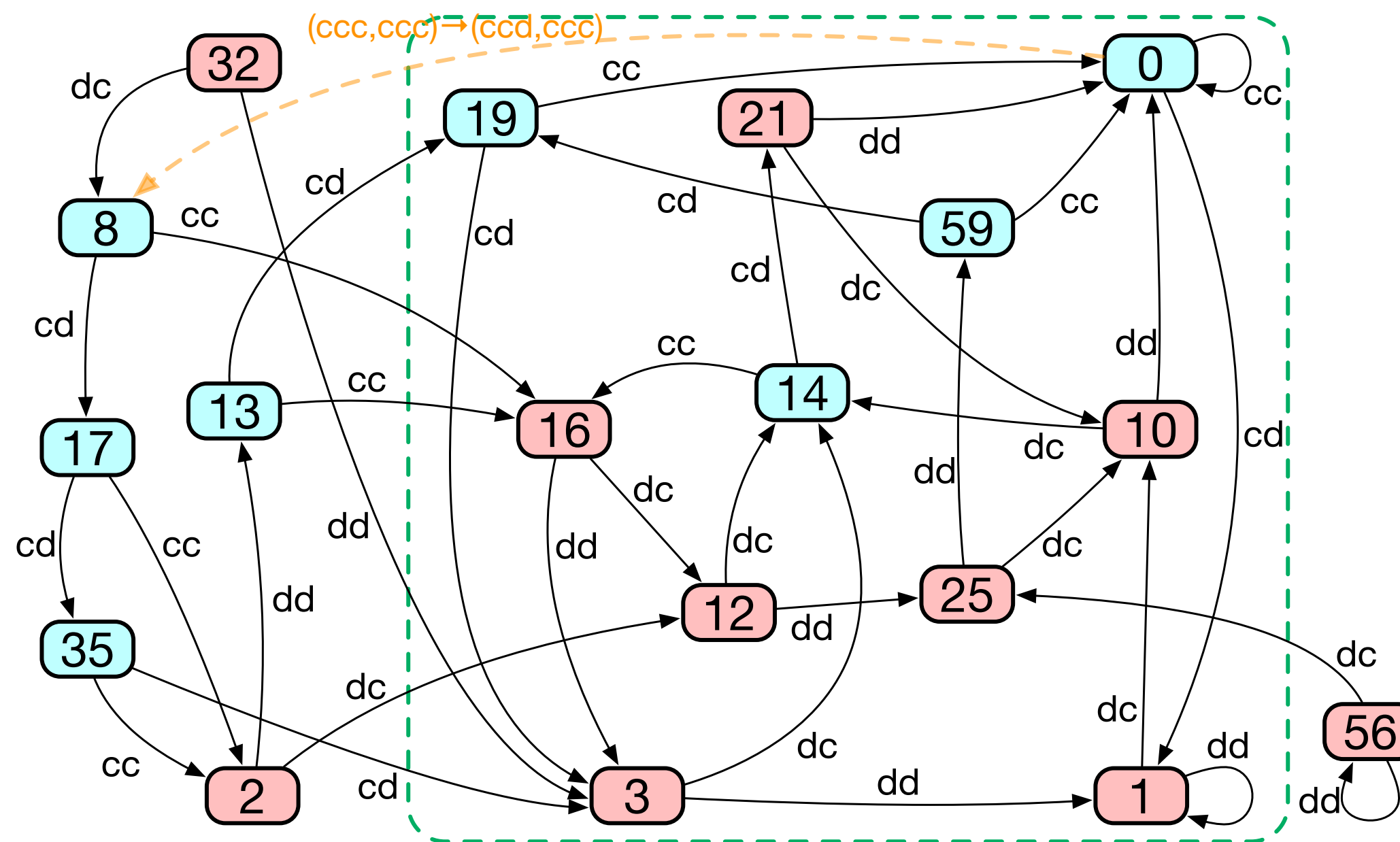


We found  
**4,261,844,305,281**  
 friendly rival strategies.





most of the strategies are not easy to interpret...



# CAPRI: a simple friendly rival

memory-3 strategy described by the five rules

$A_{t-3}A_{t-2}A_{t-1}$	$ccc$	$ccd$	$cdc$	$B_{t-3}B_{t-2}B_{t-1}$			$ddd$
	$c$	$d$	$c$	$cdd$	$dcc$	$dcd$	$ddc$
$ccc$	$^1c$	$^3d$	$d$	$d$	$^{2,4}c$	$d$	$d$
$ccd$	$^2c$	$d$	$^3c$	$d$	$d$	$d$	$d$
$cdc$	$d$	$^2c$	$d$	$d$	$^3c$	$d$	$d$
$cdd$	$d$	$d$	$d$	$d$	$d$	$d$	$d$
$dcc$	$^{3,4}c$	$d$	$^2c$	$d$	$^4c$	$d$	$^4c$
$dcd$	$d$	$d$	$d$	$d$	$d$	$d$	$d$
$ddc$	$d$	$d$	$d$	$d$	$^4c$	$d$	$^4c$
$ddd$	$d$	$d$	$d$	$d$	$d$	$d$	$^4c$

**C**ooperate at mutual cooperation.

**A**ccept punishment when you made a mistake.

**P**unish co-player and then forgive him.

**R**ecover cooperation when someone cooperated.

**I**n all the other cases, defect.

CAPRI c c c c c c

CAPRI c c c c c c

CAPRI c c c **d** c c

↖ conduct punishment

CAPRI c c **d** c c c

↙ accept punishment

CAPRI **d** **d** **d** c c c

**d** **d** c c c c

CAPRI **d** **d** **d** **d** **d** **d**

**d** **d** **d** **d** **d** **d**



# CAPRI-n: generalization to n-players game

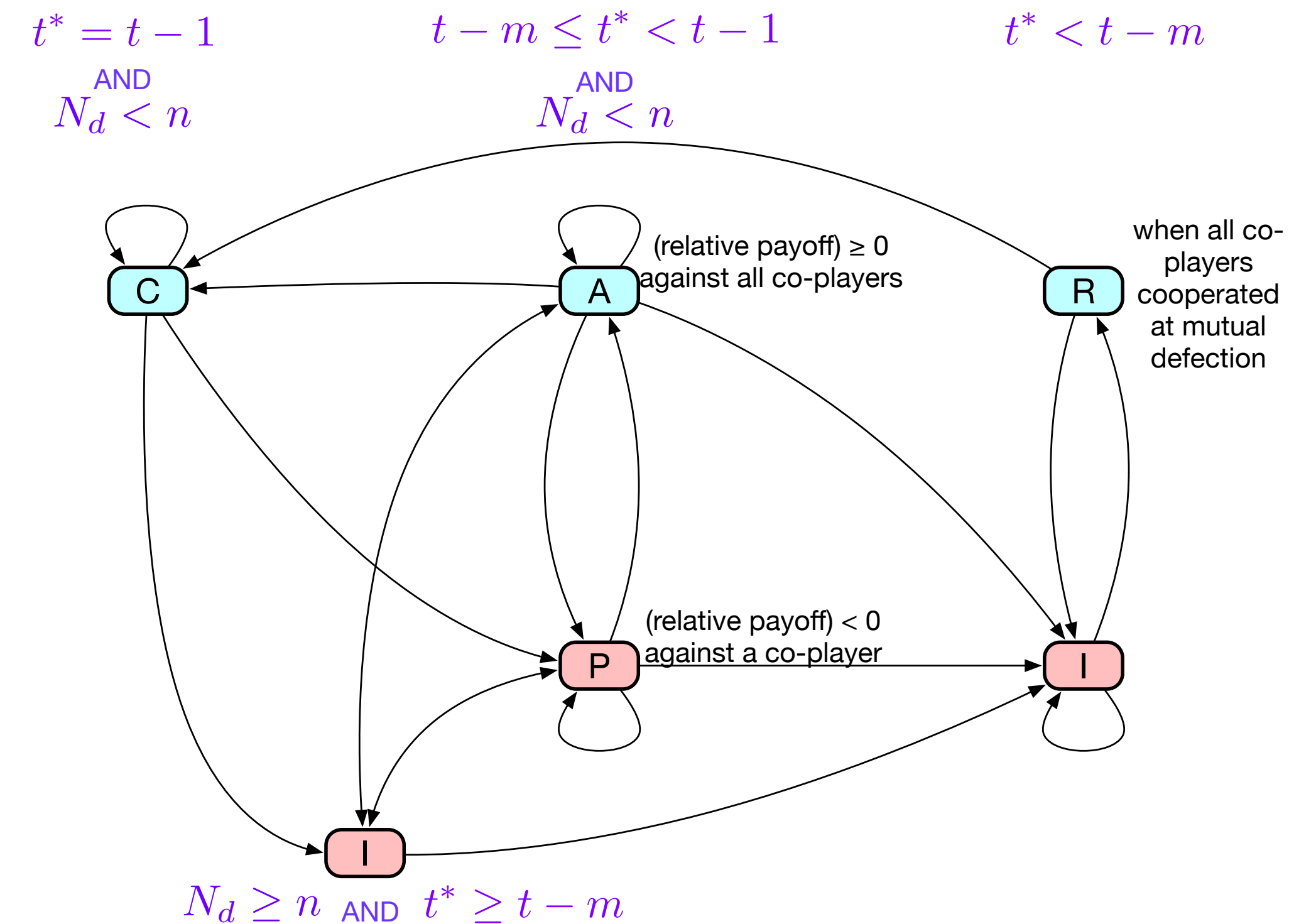
**C**ooperate at mutual cooperation

**A**ccept punishment when relative payoff is highest

**P**unish co-player when relative payoff is lower

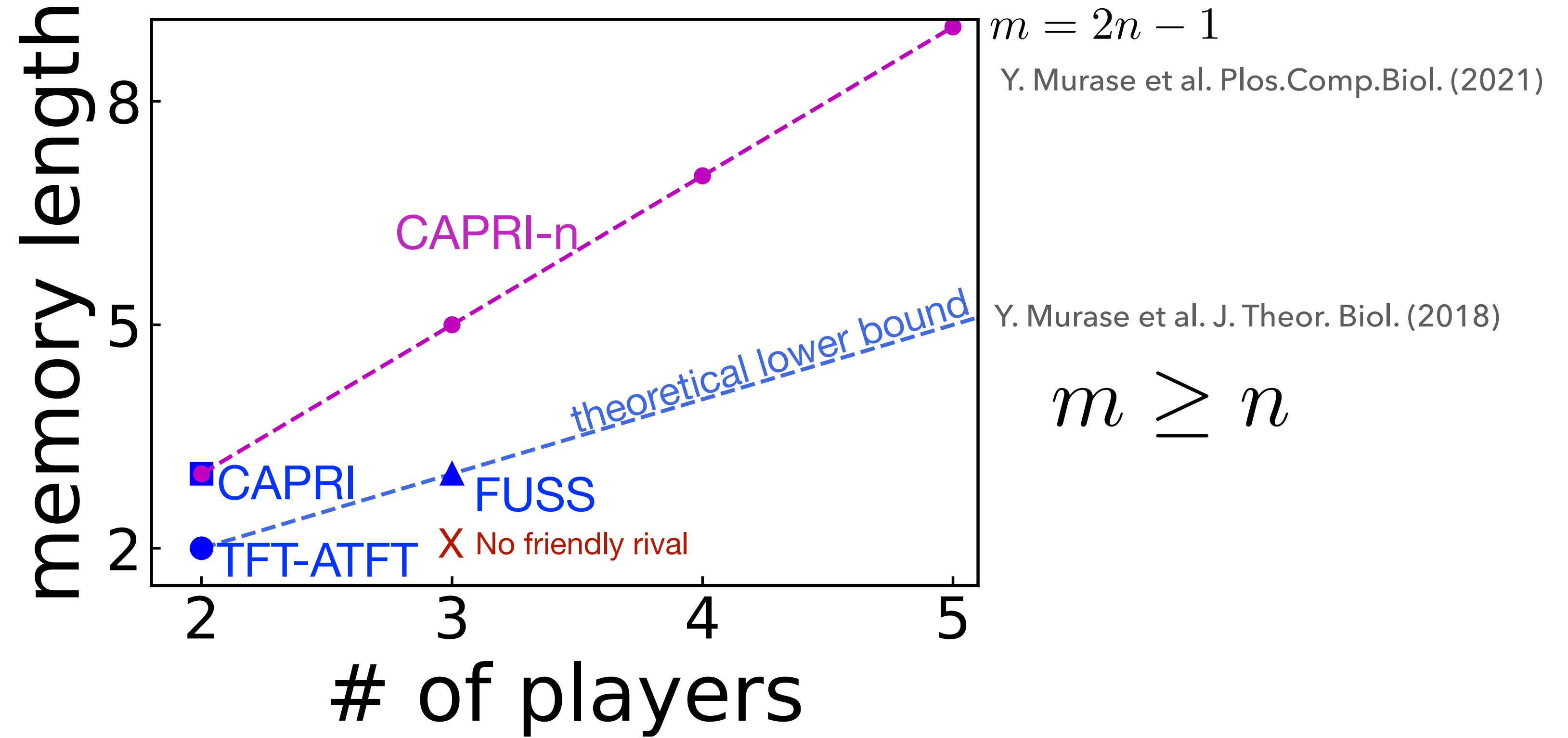
**R**ecover cooperation when the others cooperated

**I**n all the other cases, defect.



# # of players vs memory length

relationship between memory length  $m$  and the number of players  $n$



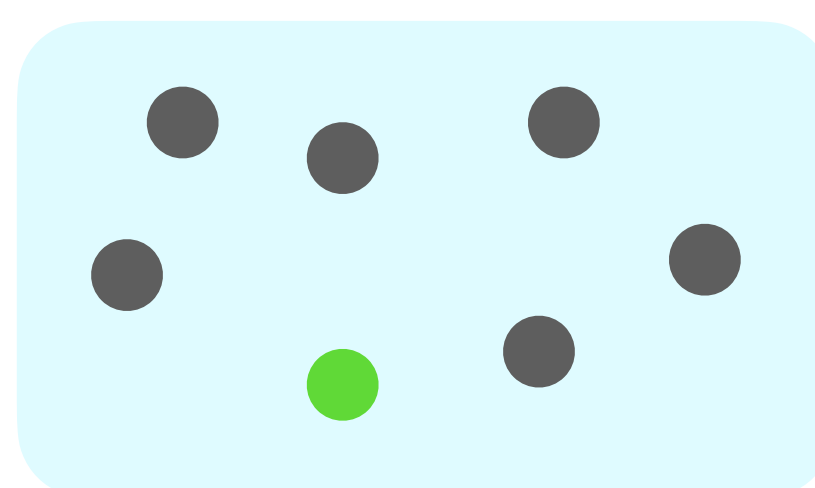
Y. Murase et al. Sci.Rep. (2020)

S.D. Yi et al. J. Theor. Biol. (2017)

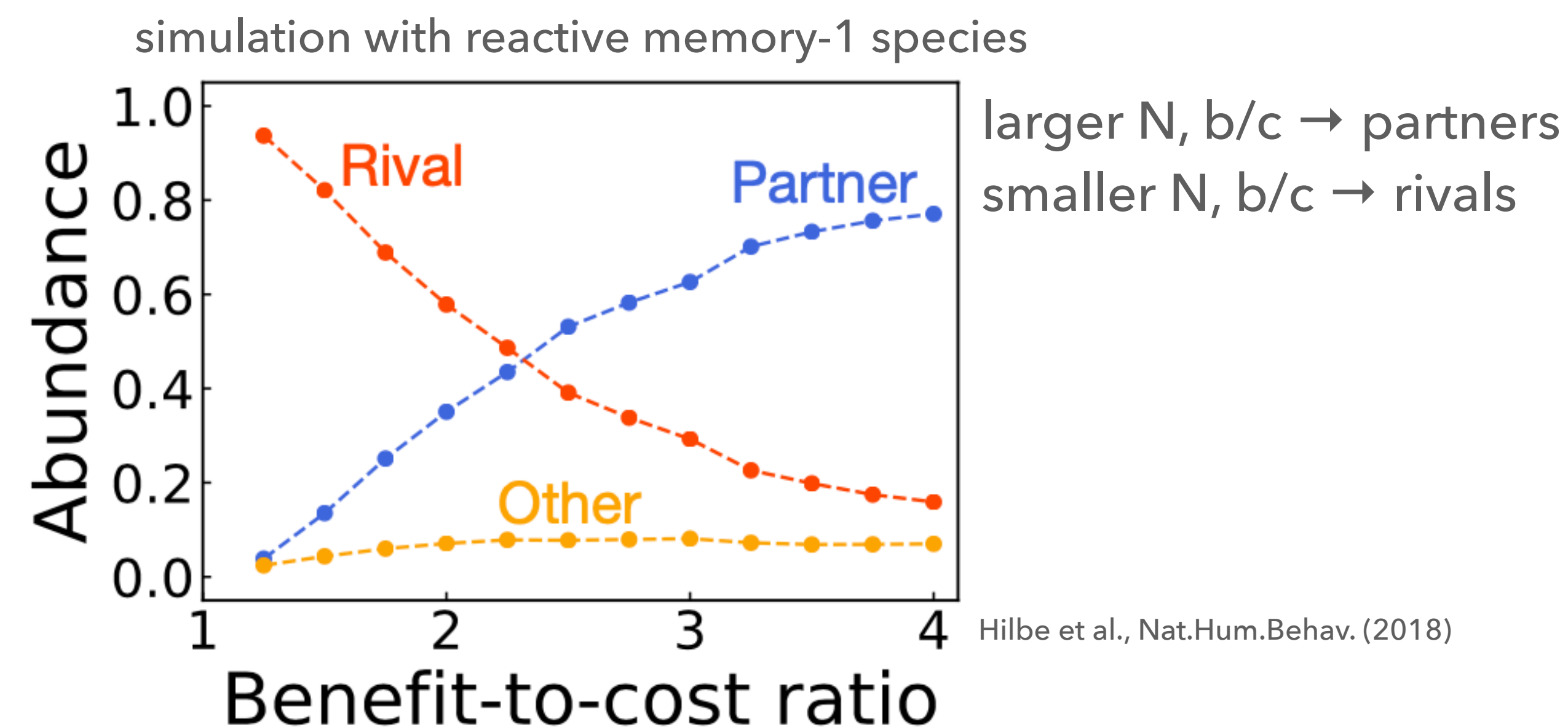
# Do friendly rivals evolve?

well-mixed population of  $N$  individuals  
 updated by imitation process

$$f_{x \rightarrow y} = \frac{1}{1 + \exp[\sigma(s_x - s_y)]}$$

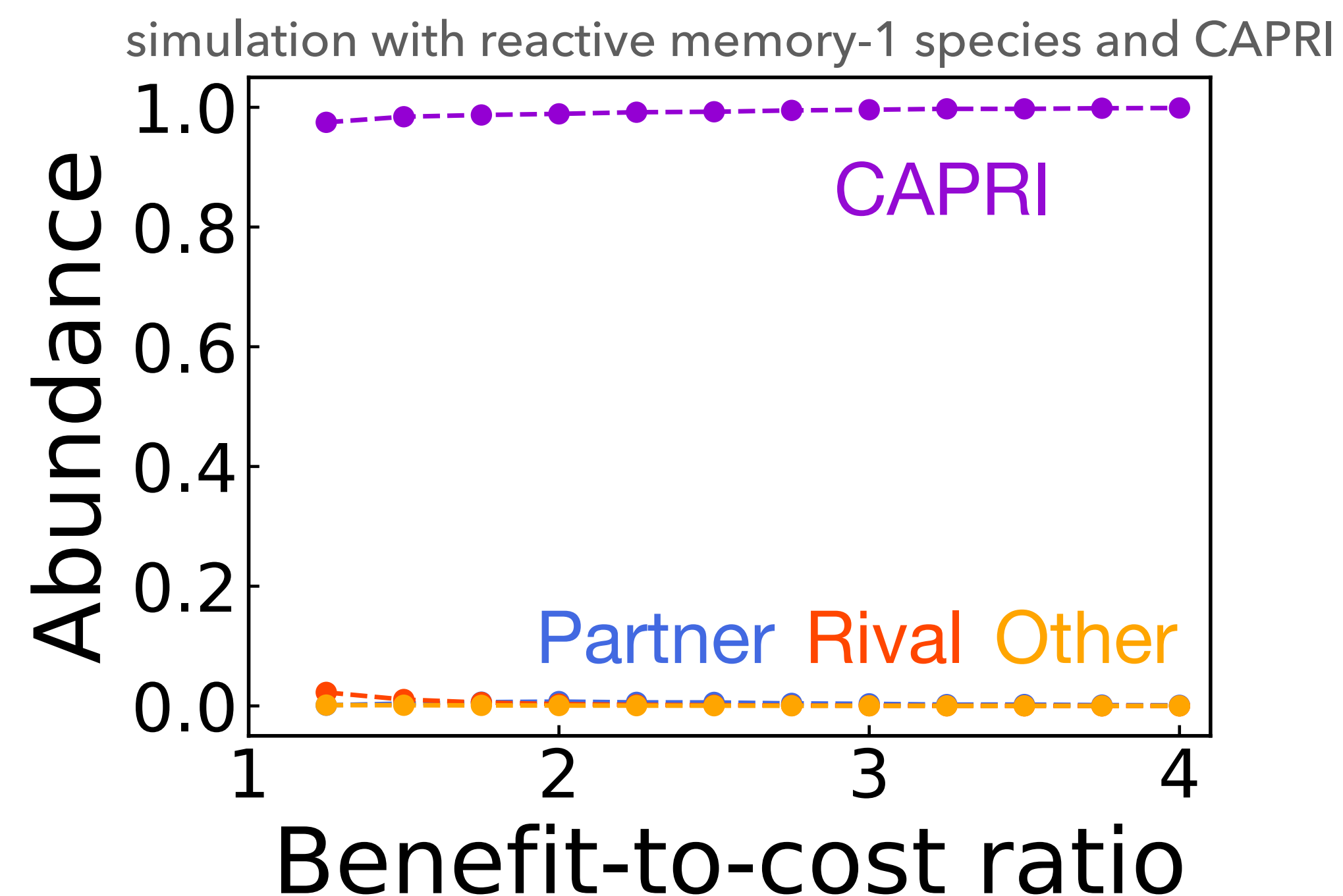


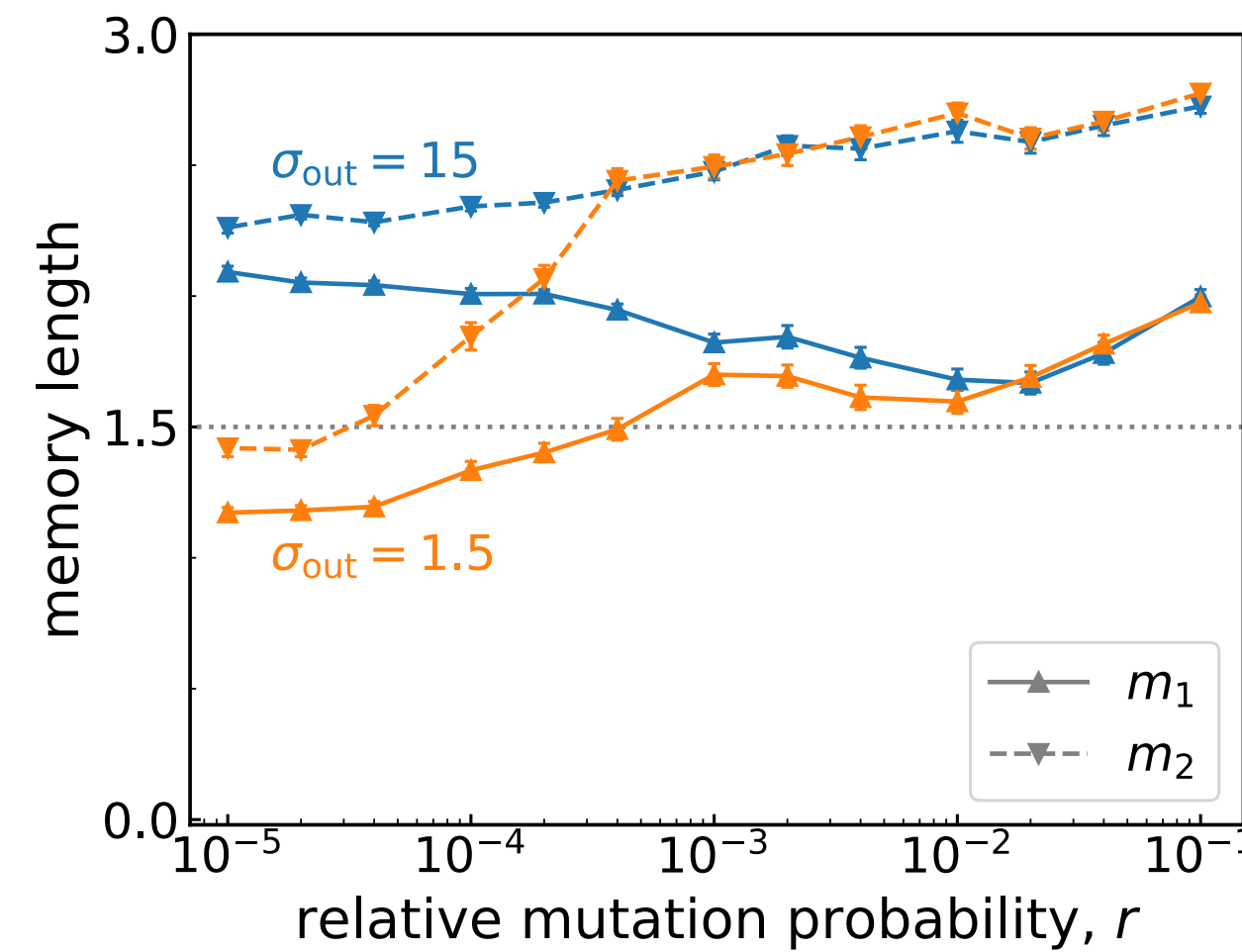
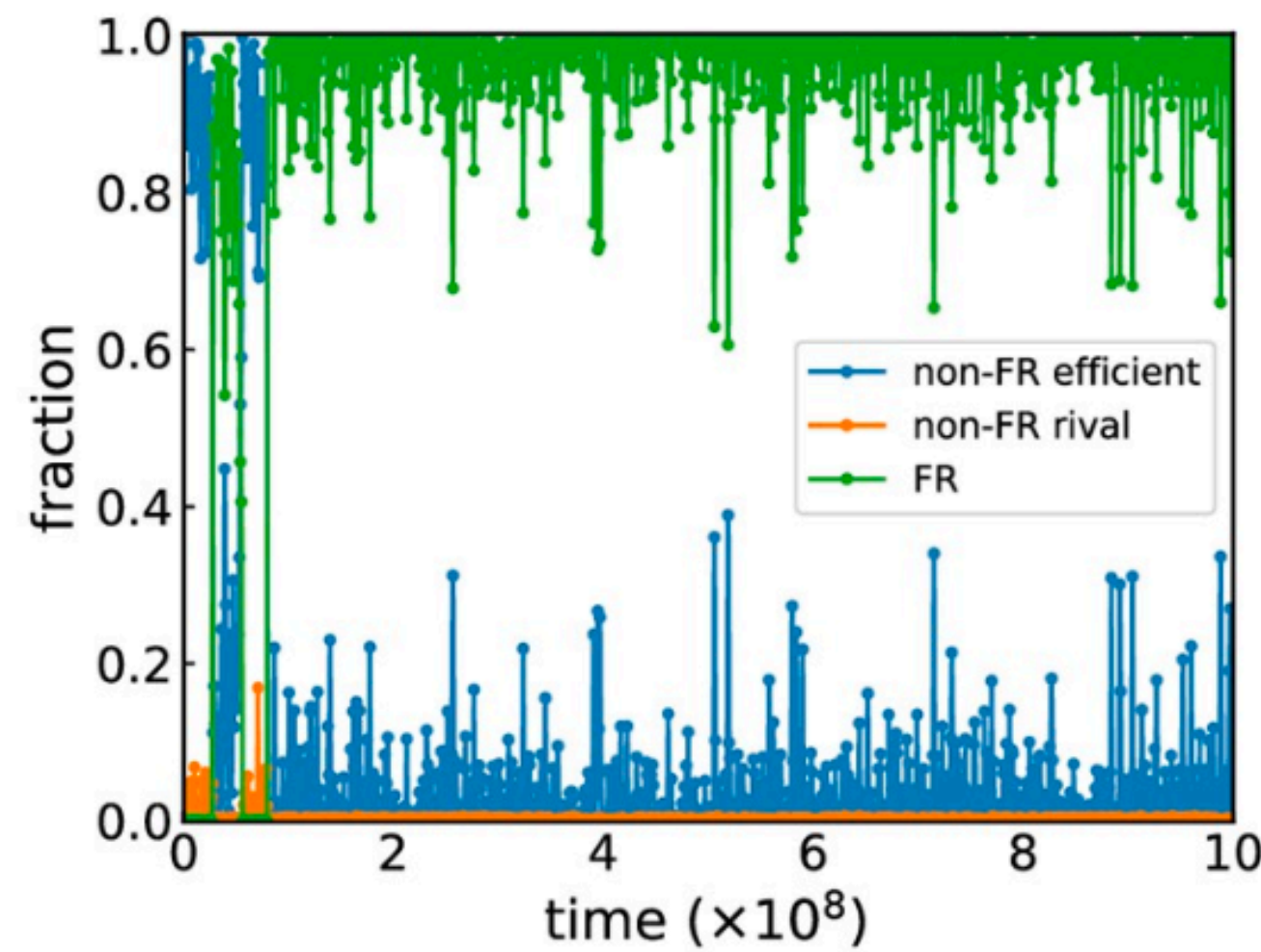
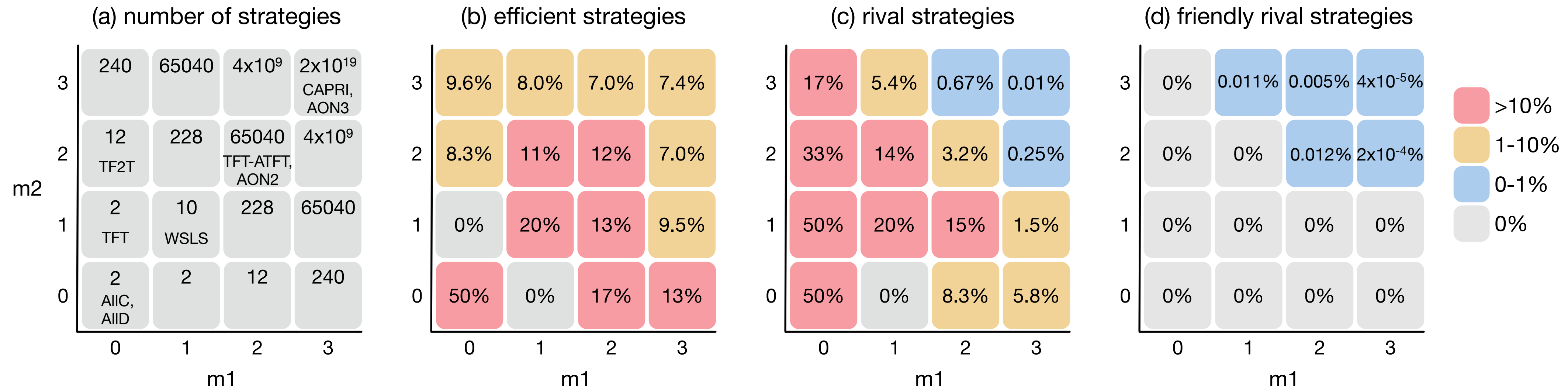
A mutant with a random strategy occasionally appears



friendly rivals are evolutionarily robust  
 for **any**  $N, b/c,$  and  $\sigma.$

$$\rho \leq \frac{1}{N} \text{ for any mutant}$$

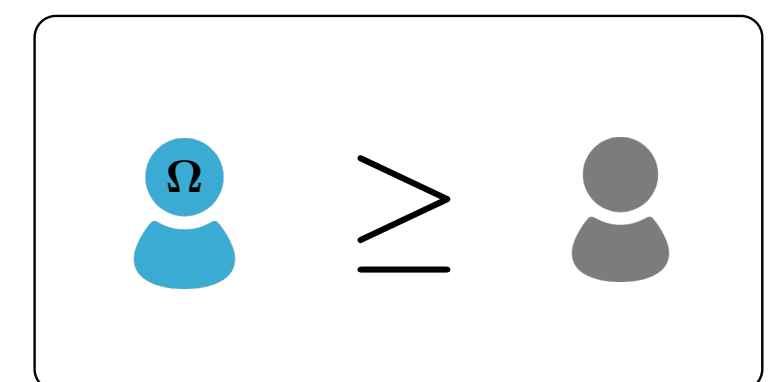
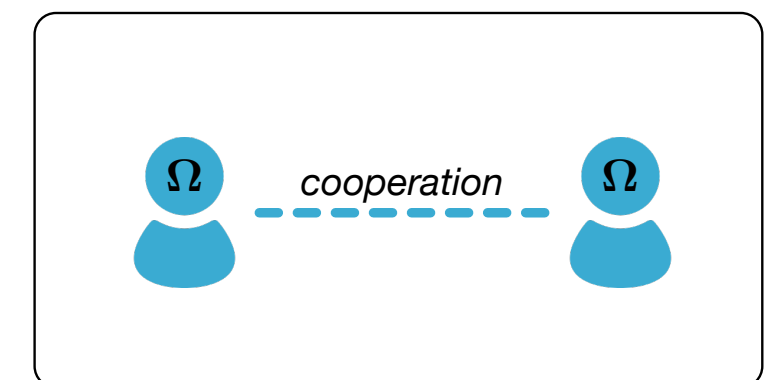
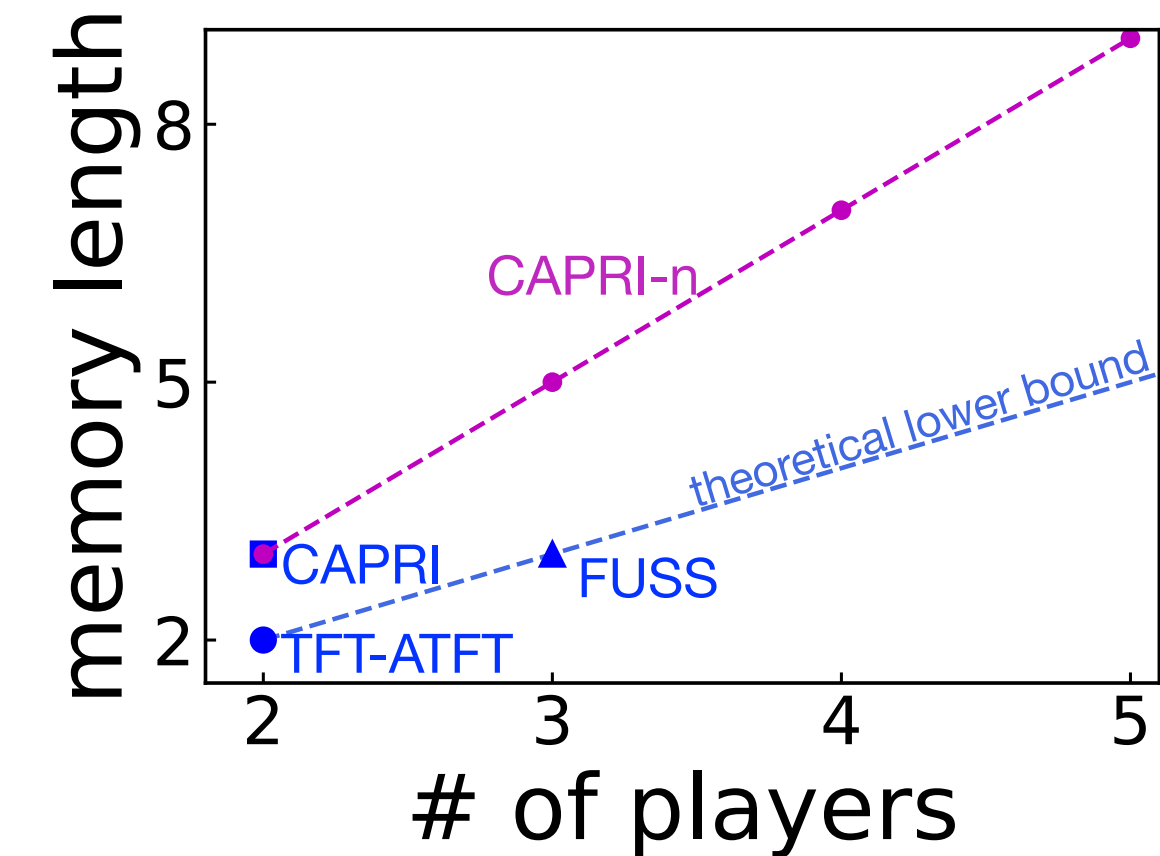




FR players remember the co-player's history better than his or her own.

# Conclusion

- We found friendly rival strategies in iterated Prisoner's dilemma and public-goods games.
- There is a critical memory lengths (strategic complexity) above which a fundamentally new class of strategies appear.
- Among them, we found **CAPRI** that is described by five simple rules. It has a number of desirable properties.
  - cooperation is robust against error (**partner**)
  - never allows any co-player to outperform you (**rivalry**)
  - **evolutionary robust** for any environmental conditions
  - exploit a broad range of strategies (AllC, WSLS), which prevents neutral drift
  - is a deterministic strategy whose actions are independent of the values of benefit or cost of the game
  - not only for PD but for the stag-hunt game and the snow-drift game
    - $(2R > T+S, T > S)$
  - generalizable to n-players public goods game





# References

- Y. Murase and S.K. Baek “Seven rules to avoid the tragedy of the commons” *J. Theor. Biol.*, 449, 94-102 (2018)
- Y. Murase and S.K. Baek “Automata representation of successful strategies for social dilemmas” *Sci. Rep.*, 10, 13370 (2020)
- Y. Murase and S.K. Baek “Five rules for friendly rivalry in direct reciprocity” *Sci.Rep.* 10, 16904 (2020)
- Y. Murase and S.K. Baek “Friendly-rivalry solution to the iterated n-person public-goods game” *Plos Comp. Biol.* (2021)
- Y. Murase, C. Hilbe, S.K. Baek “Evolution of direct reciprocity in group-structured populations” *Sci. Rep.*, 12, 18645 (2022)
- Y. Murase, S.K. Baek “Grouping promotes both partnership and rivalry with long memory in direct reciprocity” *PLoS Comp. Biol.* 19(6): e1011228 (2023)



# HPCを用いたゲーム理論の将来展望

- これまでは解析的な理論研究や実験がメイン。いかに複雑な社会現象を単純な（手で解ける）数理モデルに落とし込むか
- HPCを用いることで解析可能な問題の範囲が広がる
  - 望ましい特徴を持つ戦略を大規模に探索する（公理的アプローチ）
  - 利得の計算が簡単でない場合の解析（進化的アプローチ）
- ありうる応用
  - 人間行動、心理的傾向や文化に見られるパターンを進化的観点から説明する
  - 社会システムのデザイン。（例：オンラインシステムで評判をどのように共有するべきか）
- 現状
  - まだ黎明期。手法が系統的に確立しているわけではなく、適切な問題を見つけ、個別に計算手法を開発
  - 基本的に embarrassingly parallel。単純な並列でも取り組むべき研究課題は多く、重要度の高い課題から取り組んでいる